Techniques for the Functional Classification of Two-Chord Sequences[1]

Brent Auerbach

I. Introduction to Chordal Sequences: Ambiguity, Implied Functionality, and the Source Set of Two-Chord Patterns

This paper investigates chordal sequence, a phenomenon said to occur whenever a pattern of two or more triads is immediately restated under transposition.[2] Chordal sequences have long pervaded Western music, and theorists have long been sensitive to their unique structural and expressive attributes. Writing in 1708, Francesco Gasparini documented a large number of these chord patterns in the music of his teacher, Archangelo Corelli.[3] In succeeding generations, Joseph Riepel and Heinrich Koch began investigating sequences' syntactical and form-building capacities.[4] What may be dubbed the modern age of sequence study was launched by Francois Fétis's observation that many of these patterned motions call for suspension of the rules of common-practice harmony and voice leading.[5] Citing the "illogical progressions" found in many chordal sequences, Hugo Riemann

---

[1] The author would like to thank Edward Gollin for his helpful suggestions for

[2] The term sequence generally refers to the immediate transposed repetition of any pitch configuration, irrespective of the number of voices. It is reasonable to assume that the adjective qualifier in the phrase "harmonic sequence" was originally neutral, signifying that most vertical note arrangements in a multi-voice sequence are tertian, not that they exhibit functional-harmonic tendency. That time is long past; see Crocker (1962, 16-17) and Forte & Gilbert (1982, 85) for discussion of functionality creep vis-à-vis the term "harmony". I therefore use "chordal sequence" to refer to all tertian patterned successions, reserving "harmonic" for progressions that possess demonstrable harmonic-functional character.

[3] Hill (2005, 330). Extending a claim made by Peter Allsop (1992) concerning norms of texture in Venetian trio sonatas, Daniel Harrison traces chordal sequences to the "polychoral and echo techniques developed at the turn of the seventh century by, among others, Giovanni Gabrieli . . ." (2003, 228-229).

[4] Moreno 2000, 127-138.

[5] Fétis 1840, 164.

soon after declared them to be "not really harmonic, but melodic formulations."[6]

This view that originated with Fétis and Riemann quickly gained traction with theorists, most famously Heinrich Schenker, and remains viable to this day.[7] Yet critically, it fractured the monolithic concept of the chordal sequence. A phenomenon that once was perceived in flat, harmonic terms would henceforth be conceived as multileveled and functionally heterogeneous: a chord structure running on melodic rails.

Recognition of the inherent harmonic/melodic duality of chordal sequences was a breakthrough that regrettably remained underdeveloped for many years. In seeking an explanation for this, we can turn to Richard Bass's historical review of sequence theory. A central narrative of his account is the consistent mistreatment of this topic. On one front, Schenker, Schoenberg, and their adherents are "dismissive" toward sequences at best – e.g., through downplaying the structural significance of these patterns – and outright "disdainful" of them at worst. On another, he faults nineteenth-century harmony texts for establishing the precedent in which chordal sequences are topically segregated, hindering their integration into "music theory," writ large.[8] Much of this authorial behavior can be rationalized in terms of the duality noted above: rather than engage sequences' functional ambiguity, most theorists of the time chose to oversimplify and ignore it.

These decades of speculative and pedagogical "neglect" produced a tattered legacy. As recently as 1996, Richard Bass lamented, "There is neither consistent established terminology adequate for in-depth analyses of sequential passages nor general agreement as to classification of sequence types, or even about what constitutes a proper sequence."[9] Eighteen years later, the situation is not quite so dire. In recent years, the tools of diatonic set theory have been used to chip away at chordal sequences, exposing much of the group-mathematic content and machinery

---

[6] Bass 1996, 263.

[7] Schenker [1935] 1979, 115-117.

[8] Bass 1996, 264-65.

[9] Ibid., 265.

within that organizes and powers them. Yet despite these technological advances, Bass's pronouncement still largely resonates. A practical, comprehensive classification of even the most basic two-chord sequences, as we shall see, has indeed proven elusive. In truth, we continue to lack a clear answer to the even more fundamental question, "How do sequences function in music?" In response we find only incomplete generalizations, half-truths that well describe certain sequence families but fail to apply to others. Compounding this discipline-wide uncertainty is the outright disagreement among scholars who, even now, promote competing views on chordal sequence both in print journals and textbooks.

As this essay's classification methods will hinge on clarifying the ambiguity at the heart of chordal sequences, it makes sense to begin with discussion of that ambiguity's most prominent symptom. I refer here to the competing methods of nomenclature that currently apply to two-chord sequences. One popular way to describe the pattern shown in Example 1, exhibiting root succession D-A-B♭-F-G-D, is by traditional label. One might recognize it as an instance of the "Pachelbel Progression" or, more formally, the Romanesca schema (see Example 1a).[10]

A second labeling method that has recently gained prominence is more objective and hierarchic. Example 1b reads the same sequence as two-chord "model" that self-generates via one or more "copies."[11] The D3 [A5] label, which can be read as "Descending 3rds coordinate Ascending 5th motion," describes the distances between triad roots at two levels of organization.[12]

---

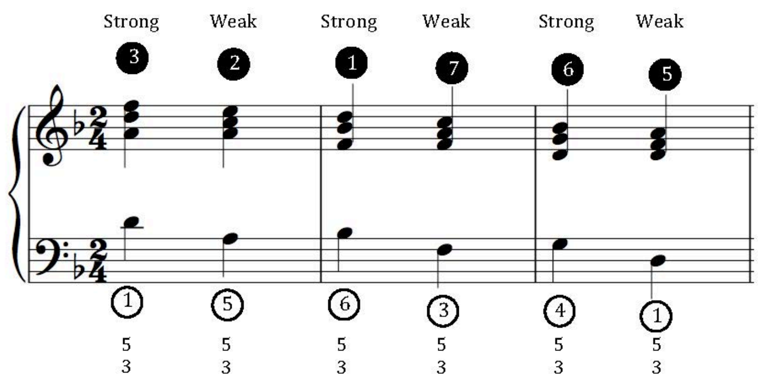
[10] The term Romanesca, from Gjerdingen 2007 (29), refers to a "six-stage" chord succession in which a "descending stepwise melody" and "a bass alternating [descending fourths and ascending seconds]" are coordinated as "a series of 5/3 sonorities" in strong-weak configuration.

[11] This view has emerged as dominant in most recent articles, e.g., Ricci 2002, Harrison 2003, and Kochavi 2008. Textbooks usually make mention of the nested patterning, however, only Laitz 2012 adopts it as an explicit pedagogic principle.

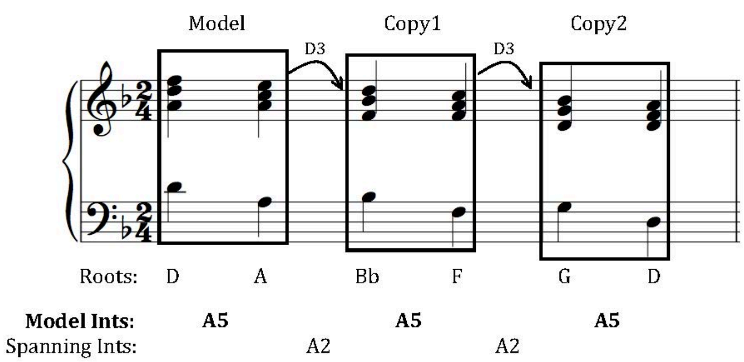
[12] In determining the root pc-interval label, the interval of transposition receives priority; the secondary status of the interior model motion is signaled by placement in brackets. This labeling procedure follows Laitz 2012, except that only the primary intervals, 5ths, 3rds, and 2nds, are allowed as opposed to their

Example 1. A well-known two-chord sequence supports traditional (a) and root pitch-class interval (b) labels.

(a) Traditional Designation: **Romanesca**[13]
Schema includes information concerning melodic and bass scale degrees, triadic inversion, and metric strength.



b. Root pc Interval Label: ***D3 [A5]***



inversional derivatives, 4ths, 6ths, and 7ths. My reasons for preferring these contextual interval classes will be made clear in Part II, below.
[13] Gjerdingen 2007, 29.

Owing to the fact that distances are calculated as directed pitch-class intervals between chord roots in mod-7 diatonic space, the term "root pc interval label" will be used to designate this naming convention.[14]

The **transpositional interval**, listed first in the label and appearing in Example 1b above the arrows connecting the rectangles, is a descending third, or D3. The **model interval**, listed second in brackets and occurring within the rectangles, is an ascending fifth, or A5. The example also gives information about the leftover, **spanning interval**. Though not an explicit labeling element, this distance will play a role in the upcoming theoretical discussion.

The plurality of nomenclature exemplified above is rooted both in theorists' deep respect for tradition and their desire to apprehend and celebrate chords as vertical versus linear structures. This flexibility, though, carries a methodological cost. We note that the situation is further complicated when multiple formulaic labels apply to a single chord series. This is shown in Example 2, Readings A and B, in which model and spanning intervals are swapped. These forms, combined now with the traditional label shown in Reading C, allow for three distinct labels to apply to a single two-chord pattern.

When put to the purpose of guiding sequence construction, the three labels are procedurally equivalent. Yet they are distinct, epistemologically. When an analyst encountering the music of Example 2 applies Reading A, she is implicitly declaring it to have a harmonic character: the descending 5th chord motion that serves as its basis is the strongest and most distinctive harmonic motion available in tonality. If she applies Reading C, the implication is that the sequence exhibits primarily contrapuntal traits.[15] Though triads

---

[14] The notes and intervals of this space are often rendered numerically. I have opted for traditional note names and directed intervals – i.e., ascending (A) and descending (D) 2nds, 3rds, and 5ths. For discussion and defense of the notion of directed pitch-class intervals as a "convenient abstraction of the sort composers regularly deploy," see Tymoczko (2008, 5).

[15] Throughout this essay, the word "contrapuntal" will substitute for the more precise term "parsimonious." This synecdoche is motivated by a longstanding trend in parlance in which highly parsimonious progressions, such as that shown

are still present, they are regarded as byproducts of the controlling contrapuntal framework. Reading B, falling between these two extremes, implies that the sequence exhibits both harmonic and contrapuntal characteristics.

My reason for bringing up the convention of plural labeling is not to denounce it. Tonality is by nature ambiguous with regard to the interplay of counterpoint and harmony, such that any attempt to describe a formation purely in terms of one or the other is doomed to fail. My point, rather, is to note two long-standing conditions in diatonic theory and, for perhaps the first time, coordinate them:

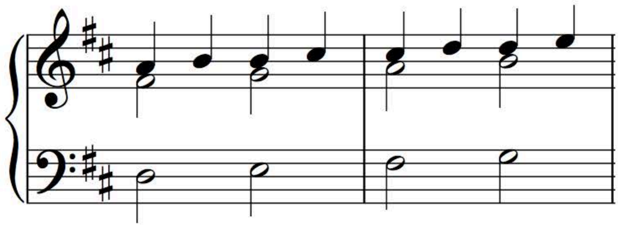
1. Chordal sequences have never been comprehensively classified to reflect their functional harmonic, melodic, and contrapuntal affinities.

2. Chordal sequences up until now have been defined in ways that alternately prioritize their various functional aspects.

in Example 2c, are explained in general as "arising out of counterpoint" and designated by iconic intervallic patterns from figured bass.

Example 2. Three legal readings apply to the same chordal sequence built of alternating, descending thirds and fifths

Reading A: A2 [D5]

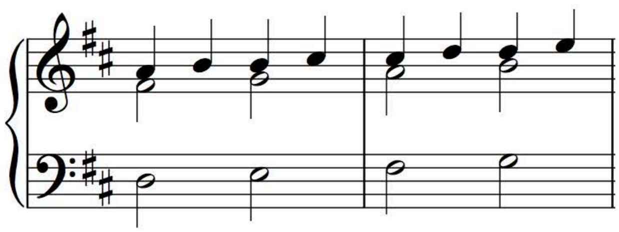


Roots: D [B E] [C# F#] [D G] E

Model Ints: **D5** **D5** **D5**

Spanning Ints: D3 D3 D3 D3

Reading B: A2 [D3]

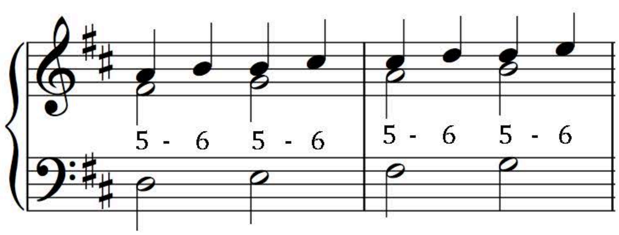


Roots: [D B] [E C#] [F# D] [G E]

Spanning Ints: D5 D5 D5

Model Ints: **D3** **D3** **D3** **D3**

Reading C: Ascending 5-6 Sequence



The way forward towards a classification system that is both multifaceted and systematic is suggested by the interdependence of these two conditions. The way to fill the longstanding lacuna of Condition 1 is to generalize and extend Condition 2.

Clarity regarding the functionality of two-chord sequences requires two things. The first is a set of techniques for consistently measuring the harmonic, contrapuntal, and melodic characters of all root motions. The second is a comprehensive set of sequences to which the metric can be applied. Regarding the latter, a useful tool for representing the total set of chordal sequences is afforded by Example 3, a table from Ricci 2002 that documents the structural properties of all two-chord sequences in mod-7 diatonic space. The comprehensive list of ordered chordal sequences appears in the far right column. With each move leftward, new equivalences group the 30 sequences into smaller classes. The 18 Unordered Sequences result from disregarding the order in which the 30 ordered sequences' constituent intervals occur. The nine Sequence Classes to *their* left result when diatonic interval classes are introduced.[16] (My use of the ~ symbol between intervals indicates directional reversal, or "zigzag" orientation, as opposed to a unidirectional one; e.g., C-E-A-C-F-A versus C-E-B-D-A-C for alternating 3rds and 5ths).

It is a mistake to think that the hierarchy among columns in Example 3 could provide a ready-made basis for a classification system rooted at all in musical practice. It cannot because the equivalences in each class are purely mathematical and do not reflect musical sensibilities. Note that the chart communicates nothing in its current state about why a select few ordered sequences have grown to prominence in the literature while others – in some cases duplicates from the standpoint of interval content – are virtually unknown.[17]

---

[16] The content of Example 3 with regard to ordered sequences and parent sequence classes corresponds directly to Tables 2 and 3 from Clough 1979 (also noted in Ricci 2002, n. 21).

[17] As further testament to this point, no previous scholar working to theorize sequence class has even attempted to relate their musical behavior to their inherent mathematical properties. Tymoczko, after listing the 18 unordered

The format of Example 3 merely suggests a framework for studying content and functionality in chordal sequences. One of the virtues of the chart is that it emphasizes model/spanning interval content. Relocating the transpositional interval to the far left temporarily frees us from interpretive distractions associated with ordered sequences, notably the question of how to parse strings of chord roots. Another of the chart's strengths is its right-to-left, progressive grouping structure, which will influence the method directly through its suggestion of two evaluative input spaces.

The Introduction now largely concluded, the body of the paper will be dedicated to processing the contents of these columns. Part II investigates the most general space of sequence-classes with the aid of a metric that objectively quantifies the total functional behavior of triadic root motions. This will lead us to group the nine sequence classes according to their harmonic, contrapuntal, and/or melodic characteristics at both surface (model) and higher (transpositional) levels. Part III will examine the 18 unordered and 30 ordered sequences, analyzing source chord strings twice to prioritize – thus keeping in balance – the alternate functional aspects of their model root motions. Following the culmination of each of these Parts with its own summary classification scheme (Examples 8 and 19), the paper will conclude in Part IV with an extended consideration of the benefits of viewing chordal sequence through the lens of harmonic/contrapuntal/melodic functionality.

---

sequences, catalogs them according to usage as measured by statistical survey of the baroque and classical literature (2011, 241). Ricci 2002, whose explicit goal is determining a classification for sequence types, surprisingly makes no use of the high-order column groupings in Example 3; further comment on this issue will appear in Part IV, below.

Example 3. Summary chart of sequence classes, unordered sequences, and ordered sequences in diatonic space, organized by interval of transposition[18]

Interval of Transposition	Sequence Classes (9)	Unordered Sequences (18)	Ordered Sequences (30) w/ sample root succession	
by 2nd	[2nd ~ 3rd]	{↑2nd, ↓3rd}	<↑2nd, ↓3rd>	C-D-B-C-A-
			<↓3rd, ↑2nd>	C-A-B-G-A-
		{↓2nd, ↑3rd}	<↑3rd, ↓2nd>	C-E-D-F-E-
			<↓2nd, ↑3rd>	C-B-D-C-E-
	[3rd, 5th]	{↑3rd, ↑5th}	<↑3rd, ↑5th>	C-E-B-D-A-
			<↑5th, ↑3rd>	C-G-B-F-A-
		{↓3rd, ↓5th}	<↓5th, ↓3rd>	C-F-D-G-E-
			<↓3rd, ↓5th>	C-A-D-B-E-
	[5th, 5th]	{↑5th, ↑5th}	<↑5th, ↑5th>	C-G-D-A-E-
		{↓5th, ↓5th}	<↓5th, ↓5th>	C-F-B-E-A-
by 3rd	[2nd, 2nd]	{↑2nd, ↑2nd}	<↑2nd, ↑2nd>	C-D-E-F-G-
		{↓2nd, ↓2nd}	<↓2nd, ↓2nd,>	C-B-A-G-F-
	[2nd, 5th]	{↑2nd, ↑5th}	<↑2nd, ↑5th>	C-D-A-B-F-
			<↑5th, ↑2nd>	C-G-A-E-F-
		{↓2nd, ↓5th}	<↓5th, ↓2nd>	C-F-E-A-G-
			<↓2nd, ↓5th>	C-B-E-D-G-
	[3rd ~ 5th]	{↑3rd, ↓5th}	<↑3rd, ↓5th>	C-E-A-C-F-
			<↓5th, ↑3rd>	C-F-A-D-F-
		{↓3rd, ↑5th}	<↑5th, ↓3rd>	C-G-E-B-G-
			<↓3rd, ↑5th>	C-A-E-C-G-
by 5th	[3rd, 3rd]	{↑3rd, ↑3rd}	<↑3rd, ↑3rd>	C-E-G-B-D-
		{↓3rd, ↓3rd}	<↓3rd, ↓3rd>	C-A-F-D-B-
	[2nd, 3rd]	{↑2nd, ↑3rd}	<↑2nd, ↑3rd>	C-D-F-G-B-
			<↑3rd, ↑2nd>	C-E-F-A-B-
		{↓2nd, ↓3rd}	<↓3rd, ↓2nd>	C-A-G-E-D-
			<↓2nd, ↓3rd>	C-B-G-F-D-
	[2nd ~ 5th]	{↑2nd, ↓5th}	<↑2nd, ↓5th>	C-D-G-A-D-
			<↓5th, ↑2nd>	C-F-G-C-D-
		{↓2nd, ↑5th}	<↑5th↓ ↓2nd>	C-G-F-C-B-
			<↓2nd, ↑5th>	C-B-F-E-B-

[18] Adapted from Ricci 2002, 12.

II. Functional Classification of the Nine Sequence Classes

In response to the noted conditions impeding sequence classification, the introduction prescribed a two-pronged solution: apply a metric evenly to a comprehensive set of sequences. We will turn to the issue of developing the first of these measures of functionality shortly. Our more pressing concern is to introduce the target space to which it will apply, namely the set of the nine sequence classes. Where the merits of this space have only been hinted at before, they will emerge more clearly through discussion of some concrete examples.

The primary obstacle to chordal sequence classification is the ambiguity whereby sequences may be alternately regarded in harmonic and contrapuntal terms. The freedom to select a 3rd-based or 5th-based model for a sample root succession such as D-B-E-C♯- is all well and good for an analyst. For the purposes of system-wide comparison, however, it will not do to compare the attributes of some sequences interpreted contrapuntally with others read harmonically. The source patterns must be processed uniformly. Our first solution for neutralizing the effects of one-or-the-other parsing is to retreat to this most general category of sequence class.

Example 4 shows how a sequence class, in this case [3rd, 5th], contains multiple ordered sequences depending on how the central root string is read. The common attribute of the four ordered sequences is the aggregate interval content. For this class, all motions encompassing three adjacent roots will traverse in total one 3rd and one 5th.

Example 4. Derivation of four ordered sequences from a sequence class

Class [3rd, 5th]

< ↑3rd, ↑5th> ⟶ ⟵ < ↓5th, ↓3rd>

C E B D A C G B F A E G D F

< ↑5th, ↑3rd> ⟶ ⟵ < ↓3rd, ↓5th >

All that is needed is a means for determining the full functional character of root motions by step, 3rd, and 5th. Once these values are known, applying them to an unordered sequence class will describe the total functional character of all two-chord motions that stem from it. It is simply a matter of running the calculation twice, one for each root interval, and summing the results within each category.

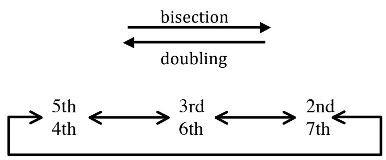
The design of the metric used for classification here in Part II will take Clough 1994 as a point of entry. He describes an important property of mod-7 diatonic space as follows: any interval except the unison produces a new type when doubled. As the two-headed arrows of Example 5a indicate, this principle applies in reverse as well, allowing for the even bisection of any interval. For example, a second is composed of two equal fifths.[19] Clough's insight is valuable first for what it reveals about the three classes of root motion, which is that each type infinitely nests the others. This corollary is illustrated in Example 5b by means of a movable wedge that shows the distribution of mixed interval characteristics. The primary interval character is given at the vertex, with subsequent interval characters growing more diffuse as one moves by degree to the right.

The further advantage of Clough's finding is that it is readily extensible, as the three essential characteristics entwined with chordal succession in general – harmony, melody, and counterpoint – are mapped on to the three types of root motion.

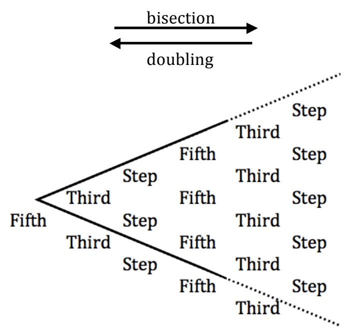
[19] Clough's formal demonstration of this property involves applying the M2 operator ("extraction of every other element") to the equal-interval subspaces numbered s1 through s6 (1994, 231-235). The two closed circuits that result, [s1-s2-s4] and [s6-s5-s3], are analogous to the interval loops shown in Example 5b.

Example 5. Some properties of the diatonic set as noted in Clough 1994

a. Chart of nested intervallic relations. Leftward arrows indicate intervallic doubling and rightward arrows even bisection.

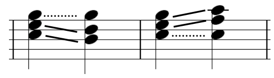


b. Doubling/bisection chart for the three interval types. The movable wedge shows the nested intervallic characteristics for any starting interval class, in this case third-ness.

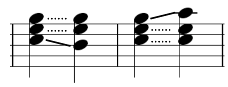


Example 6. Characterization of the three root motion types, along with information about abstract voice-leading distance (VLD) and common-tone retention (CT)

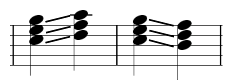
a) Motion by fifth is maximally **harmonic :**
minimum VLD = 2, CT = 1



b) Motion by third is maximally **contrapuntal**:
minimum VLD = 1, CT = 2



c) Step motion is maximally **melodic**:
minimum VLD = 3, CT = 0



Example 6 models the three fundamental root motions. For each, information is included about common tone retention and total voice-leading distance (VLD) as measured by summing the absolute-value motions of all voices ("Taxicab metric"). Based on the established principle that the most directed chord progressions result from fifths, this type of motion is deemed "maximally harmonic".[20] In Line a, root motion by fifth is shown to exhibit a minimum VLD of 2, with one common tone potentially retained. Motion by third in Line b is marked by the highest degree of parsimony. Its minimum VLD is 1, with two common tones retained. Typically experienced more as a chordal transformation than a functional, directed progression, third motion is deemed "maximally contrapuntal," a character that in this context is furthest removed from "maximally harmonic."[21] Motion by

[20] Aldwell and Schachter are unequivocal on this point, noting that "harmonic progression is organized by the 5th relationship." Relying on insights that date back to Rameau, the authors further explain: "The 5th is the first 'new' tone in the overtone series" and "in triadic music, [it] is uniquely able to define the root or fundamental tone of a triad" (2003, 60).

[21] It may seem troubling to assert that a triad succession conceived in the abstract—e.g., a stepwise ascending pattern of C-D or G-A—exhibits consistent

second, which has a minimum VLD of 3, is characterized in Line c as "maximally melodic" on the basis of all its voices moving by step.

Example 6 only assesses the dominant character of each root motion. To obtain a fuller picture for any motion, we must measure its remaining two aspects. This is achievable through a quantification and ranking scheme of the type provided in Example 7, in which all three functional characteristics are measured on a scale between 0 and 2.

Example 7. Quantifying the functional characteristics of each root motion type

	Harmonic Ranking (degree of root close-ness by 5th, 0-2 scale)	Contrapuntal Ranking (# of CTs retained)	Melodic ranking (potential stepwise motion, 0-2 scale)
5th motion:	2	1	1
3rd motion:	0	2	0
2nd motion:	1	0	2

The rankings in the harmony category (left column) reflect how immediately fifth-like a root motion is. Fifth motion is by definition accorded the highest score, 2. Third motion, being two degrees removed from fifth-ness, gets a score of 0. A root second motion, which is more directly composed of two fifths, is intermediately valued at 1. In the center column, a root motion's contrapuntal quality – essentially, its degree of parsimonious voice leading – is valued directly in terms of the number of common tones retained. Motion by third is therefore maximally contrapuntal, whereas motion by fifth is partially contrapuntal and motion by second is minimally so. Last, in the category of melodic quality, a root motion may result in one, two, or three voices

---

degrees of harmonic and contrapuntal behavior in all contexts. I argue here that such claims can be reasonably made, provided it is kept firmly in mind that they are contextual. Great care will be taken to illustrate that the default characterization of any root motion is subject to revision as further information concerning its inversional and pitch voice-leading disposition in specific contexts comes to light.

moving by step between triads. The scaled values of 1, 0, and 2 are assigned accordingly to fifth, third, and second motion.

To take an example involving class [3rd, 5th], the root motion by 3rd yields values of 0, 2, and 0 in the harmonic, contrapuntal and melodic categories. The other motion by 5th yields values of 2, 1, and 1. The two motions combined create the 2, 3, and 1 values shown in Line 7 of Example 8. In cases where the model contains a duplicate motion, such as [2nd, 2nd], [3rd, 3rd] and [5th, 5th], the interval is scored twice. Example 8 shows the results of applying this procedure to the nine sequence classes. To aid recognition, the most familiar ordered sequence nicknames are included next to the classes from which they derive. Preliminary analysis of the data appears in the form of circles drawn around maximal values, 3 and 4.

Example 8. Sequence class characters scored according to the metric from Example 7

Seq. Class	Nickname	Harmonic character	Contrapuntal character	Melodic character
1. [2nd, 2nd]	parallel6/3s	2	0	(4)
2. [2nd, 3rd]		1	2	2
3. [2nd ~ 3rd]		1	2	2
4. [2nd, 5th]	Romanesca	(3)	1	(3)
5. [2nd ~ 5th]	4-5-1 chain	(3)	1	(3)
6. [3rd, 3rd]	circle of thirds	0	(4)	0
7. [3rd, 5th]	5-6 sequences	2	(3)	1
8. [3rd ~ 5th]	D3 w/applied Vs	2	(3)	1
9. [5th, 5th]	circle of fifths	(4)	2	2

Certain affinities and trends among the classes are immediately discernable. The first, which is somewhat self-evident, is that there is one sequence class each that is maximally harmonic, contrapuntal, or melodic. The three classes fulfilling these roles are [5th, 5th] in Line 9, [3rd, 3rd] in Line 6, and [2nd, 2nd] in Line 1;

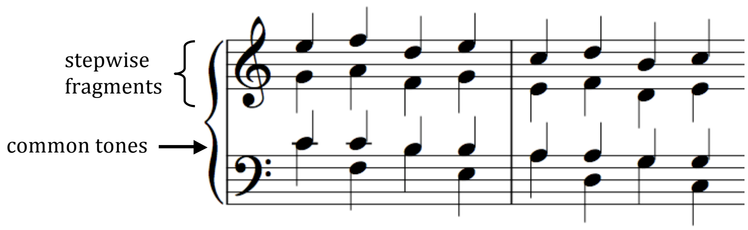
i.e., the circles of fifths, thirds, and seconds. Some new information emerges here about them, however. Sequence class [2nd, 2nd], beyond possessing an explicitly melodic character, also exhibits subtly harmonic traits. Its value of 2 in this domain makes sense in light of the fact that stepwise root progressions frequently participate in harmonic chord progression, particularly in ascent (e.g., I-ii, iii-IV, IV-V, V-vi, vii^{o}-I, and vi-V). A vivid illustration of this property is found in Beethoven's Piano Sonata, Op. 31, No. 2 ("Tempest"), movement I, mm. 29-37, where an upward march of six-three triads creates a functional progression in the key of A minor: iv^{6}-V^{6}-VI^{6}-vii^{o6}-i^{6}.

Harmonic class [5th, 5th] exhibits a hybrid quality that also enfolds melodic and contrapuntal aspects. To contextualize this result, one may think of the accompanying "2" values in the lowest line of Example 8 as documenting additional potential voice-leading information about the circle of fifths. For example, the "textbook" version of the sequence as depicted in Example 9 has three of its four voices built of melodic upward-stepwise motions or common-tones. In contrast to [2nd, 2nd] and [5th, 5th], the circle of thirds in Line 6 can be viewed as a purely contrapuntal gesture.

Another noteworthy result appears in conjunction with sequence class [3rd, 5th] (Line 7 of Example 8), which is the source for all 5-6 sequences. The present computation favors the interpretation of this sequence class as contrapuntal, a finding that has potential analytical applications. In examining a candidate sequence that is composed of alternating, unidirectional 3rd and 5th root motion (e.g., C-A-D-B-E-C-), the results in Example 8 may be cited in favor of parsing it to prioritize 3rd model motion, making the A2 [D3] designation preferable to A2 [D5]. One should of course not go too far here and mistake this preference for 3rd-based models for an ironclad rule. In many cases, musical context will indicate the alternate parsing. The results in Example 8 are most relevant to purely diatonic sequences. The injection of chromatic elements, as will later be seen, increasingly encourages fifth-based interpretations through the invocation of applied dominant harmony.

In seeking out a grander theoretical claim, we note that the difficulties routinely faced in attempting to pigeonhole sequences according to prototype and function are predicted by the mixed-functional shading results of Example 8. The inherent ambiguity of sequence class [3rd, 5th] is indicated by the close values in the harmonic and contrapuntal categories, 2 and 3. Next, consider class [2nd, 5th], from which the Romanesca derives. The values in Line 4 indicate that this sequence class is equally harmonic and melodic. This result may surprise readers accustomed to viewing this pattern in harmonic and/or contrapuntal terms as befits its usual labels, D3[A5] and "Descending 5-6." It makes sense, though, in light of this sequence's rich melodic pedigree. As Example 10 illustrates, stepwise motion is a hallmark of the Romanesca in nearly all of its common settings.

Example 9. Typical realization of sequence class [5th, 5th]



Example 10. Romanesca realization of sequence class [2nd, 5th]

The present, flexible view of root motion functionality is highly preferable to approaches that peremptorily declare certain two-chord sequences to be either harmonic or contrapuntal in nature.[22] It is further useful in illuminating the separate matter of why common practice composers tended to avoid certain chordal sequences. In the same way that maximal values congregate around the most popular sequences, minimal values group in patterns as well. Lines 2 and 3 in Example 8 only contain 1s and 2s, indicating an overall nondescript quality for classes [2nd, 3rd] and [2nd ~ 3rd]. These numbers under present valuation indicate dubious compositional value. As predicted, the sequences deriving from these classes are almost completely absent from tonal literature.[23]

The chart in Example 8 furnishes the basis for our first functional sequence classification. The pattern of distribution of maxima suggests organization into five larger groups. Lines 1, 6, 7, 8, and 9, each featureing a single maximal value, support the establishment of sequence classes on the basis of predominantly harmonic, contrapuntal, and melodic character. Lines 4 and 5, containing two maxima, indicate the presence of a fourth category that is equally harmonic and melodic. The remaining lines without maxima are indicative of a leftover category marked by the absence of any strong character.

Example 11 illustrates the five-category organization. Members in each category are further arranged according to higher-order transposition interval. For example, in Line 4, the higher order "2nd" is appended to sequence class [3rd, 5th], in contrast with the "3rd" value placed next to class [3rd ~ 5th] in Line 5. This information is accounted for in the enhanced functional designations given in the center column. For each, the x / y format indicates first, its character at the surface level as determined earlier, and second, its character resulting from transposition by 2nd, 3rd, or 5th.[24]

---

[22] Of the two shadings, authors are far more likely to argue for the primacy of the contrapuntal view over the harmonic one (Forte and Gilbert 1982, 83-102 and Damschroder 2006, 258).

[23] In Tymoczko's statistical survey, three of the four unordered sequences built from these intervals are given the lowest informal ranking of "very rare." The remaining one, sequence <↑2nd,↓3rd>, is evaluated as only slightly more prevalent, receiving the still somewhat dubious ranking "exists" (2011, 241).

[24] The decision to terminate functional characterization at the second hierarchical level is in a sense arbitrary given that all intervals infinitely nest themselves. It is

Example 11. Sequence classes organized in five categories by model and then transposition character (superscript value)

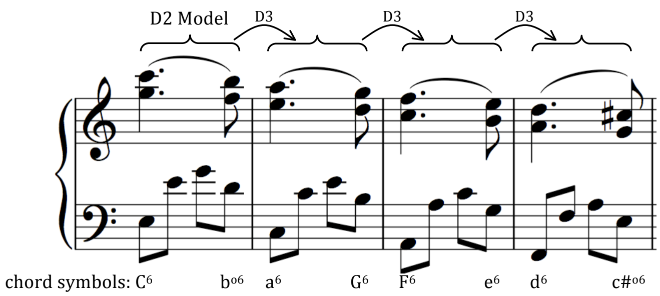
Seq. Classes	First- / Second-level characteristics	Sequence Class Category
1. [2nd, 2nd]^{3rd}	melodic / contrapuntal	Melodic Sequence Class
2. [2nd, 5th]^{3rd}	M&H / contrapuntal	Melodic&Harmonic Seq Classes
3. [2nd ~ 5th]^{5th}	M&H / harmonic	
4. [3rd, 5th]^{2nd}	cont / melodic	Contrapuntal Sequence Classes
5. [3rd ~ 5th]^{3rd}	cont / cont	
6. [3rd, 3rd]^{5th}	cont / harmonic	
7. [5th, 5th]^{2nd}	harmonic / melodic	Harmonic Sequence Class
8. [2nd ~ 3rd]^{2nd}	none / melodic	Leftover Sequence Classes
9. [2nd, 3rd]^{5th}	none / harmonic	

Our discussion of the five sequence class categories begins with the uniform-interval types. In two of the three cases, a single sequence class represents the entire functional category. [5th, 5th] in Line 7 is the sole harmonic sequence class, and [2nd, 2nd] in Line 1 is the only melodic sequence class. Even here, some important upper-level characteristics emerge that should be acknowledged. Sequence class [5th, 5th] is inherently harmonic at the surface level and – in accordance with Fétis's claim – melodic at the next-higher transpositional one. This hardly constitutes new information: these sequences are already well known by their root pc-interval labels, D2 [D5] and A2 [A5]. Carrying over this finding to another single-interval class, we see that a circle of seconds—typically realized as a parallel six-three succession—manifests at the next highest level as a contrapuntal, thirds-based phenomenon. Example 12, taken from the finale of Mozart's Piano Sonata in A

not nonsensical, however. Certain longstanding consistencies in sequence nomenclature and pedagogy indicate Western musicians' strong preference for conceptualizing sequences in two-level terms.

minor, K. 310, illustrates the dual stepwise and thirdwise attributes of this sequence class.

Example 12. Mozart Piano Sonata in A minor (K. 310), mvt. 3, mm. 37-40



The circle of thirds, [3rd, 3rd] in Line 6, exhibits harmonic character at its second level when partitioned into two-chord units. The passage given in Example 13 from Beethoven's Third Piano Concerto, finale movement, illustrates how one can hear a descending series of thirds as a larger, fifth-based progression.[25]

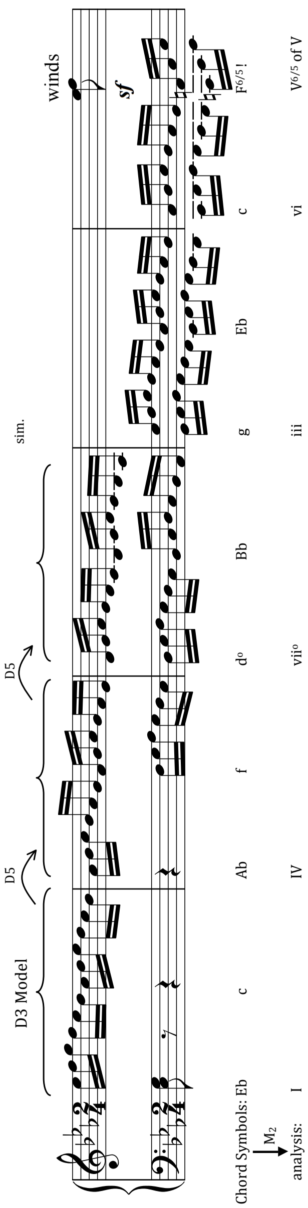
The circle of thirds is inherently a contrapuntal sequence class; however, in a remarkable asymmetry, it is not the only one of its kind. In Example 11, it is grouped with two others with second-order transposition patterns that complement it. In addition to [3rd, 3rd]^{5th}, there is the [3rd, 5th]^{2nd} class, which provides the source material for all 5-6 and 6-5 sequences. This class is melodic in the sense that it transposes by step at the larger scale, hence its contrapuntal/melodic designation. The last of the three is [3rd ~ 5th]^{3rd}, a construction used infrequently in diatonic music that is curiously contrapuntal at both the surface and upper levels.[26]

---

[25] The "harmonic" reading I-IV-vii^{o}-iii-vi, derived from applying M2 to the surface chord root succession, is supported by the sforzando marking in m. 95 that gives emphasis to a chord (F$^{6/5}$, or V$^{6/5}$ of V) that would not normally receive it due to weak-beat placement. (This sudden emphasis draws attention to the violation of the previously-established harmonic rhythm as the descending fifth motion C-F now takes up only one beat.)

[26] This class often manifests in chromatic form with harmonic function as a D3 [D5] sequence with applied dominant chords, e.g., C-E-a-C-F-A-d-etc.

Example 13. Beethoven, Piano Concerto in C Minor (Op. 37), movt. 3, mm. 91-95



The leftover classes will be excluded from consideration here. This leaves the most unique set of sequence classes, those that exhibit jointly melodic and harmonic characters at the surface. Class [2nd, 5th] in Line 2 is the source of the Romanesca, whose hybrid nature was noted above. Because its model transposes by sixth or third upon repetition, the upper-level function of this sequence is contrapuntal. The other member of this category is sequence class [2nd ~ 5th] in Line 3. None of the four ordered sequences deriving from this class appears regularly in its pure diatonic form; however, a chromatic variant of A5 [D5] is well known. It is the "4-5-1 chain" shown in Example 14, a common modulating pattern created through successive joinings of a functional IV-V-I progression. This usage makes sense in light of the fact that the two internal motions combine to create transposition by fifth; the sequence class necessarily has harmonic function at the higher level.

The chart in Example 11 yields new information about the compositional function of chordal sequences by opening a window onto the complexity of their inherent characters. For speculative theorists, it offers a thorough and objective means for classifying chordal sequences. For analysts, it provides information that can enrich hermeneutic interpretation. In seeking to account for the role of specific chordal patterns in works, musicians can move beyond vague generalizations about sequences' "circular" or "wandering" nature. In their place, they can call upon a more refined technical vocabulary that treats chordal sequences dynamically, as agents that impart specific sonic qualities to music. This enables analysts to associate contrapuntal sequences with contrapuntal gestures occurring elsewhere, rather than walling them off as a separate category of event.

Example 14: Modulatory 4-5-1 chain from Beethoven's Symphony No. 4 in B-flat (Op. 60), movt. 3, mm. 35-49

At the same time, despite its benefits, the first classification scheme exhibits a major shortcoming. It is too general in that it pertains only to the nine sequence classes. This limits our abilities to make comparisons among the 18 unordered and 30 ordered sequences enfolded within. For example, when selecting any two classes at random for comparison, the chart in Example 11 will return information only if they derive from separate classes. In response to the need for a more exacting mode of classification, the following section develops and applies another kind of functional analysis to the entries appearing in the two remaining columns of Example 3.

III. Functional Classification of the 18 Unordered and 30 Ordered Sequences

In designing our classification for ordered sequences, we first turn our attention to the 18 unordered sequences appearing in Example 3's third column. In Part II we neutralized the impact of root interval order by retreating to the left to the sequence class column. In Part III, we will directly engage the issue of root interval in a rigorous, consistent manner.

The solution advocated here is to counteract parsing preferences by processing all of the unordered sequences to separately accentuate their harmonic and contrapuntal aspects.[27] Each time the procedure runs, it polarizes the 18 unordered sequences into 18 ordered sequences of like character, allowing for direct comparisons. The harmonically-shaded sequences will be derived first, followed by the contrapuntally-shaded sequences. After internal affinities are noted for each independent set, the

[27] It is no accident that melodic, or 2nds-based, characteristics are devalued in this method. In contrast with motion by fifth and third, each of which has convincingly served as a conceptual basis for tonality—see Rameau [1722] 1971 and Tymoczko 2011 (226-238)— motion by second has never been called on in this theoretical capacity. In suggesting extensions to this classification method on pages 152-55, I note conditions that might encourage future investigators to experiment with giving increased weighting to 2nds-based models.

results will be collated into a comprehensive ordered sequence chart.

The procedure is as follows:

1. The two intervals of the unordered sequence are considered. When different, priority is given to the one exhibiting stronger harmonic or contrapuntal character according to the valuations given in Example 7 *as relevant to the desired functional shading*. If the two root motions are identical, either may serve as the model interval.

2. The characteristic interval establishes the content of the model progression. The resulting ordered sequence is named both in its descending and equivalent ascending forms.

Let us take as an example the unordered sequence {↑2nd, ↓3rd} with the following root series: C D B C A B G A. Conceived harmonically, the stepwise motion is privileged over (outscores) the third motion. An "even" parsing of these roots produces the descending form C-D, B-C, etc; the ascending form is given by reading the string in reverse, A-G, B-A, etc. The two forms of the harmonic sequence are thus D2 [A2] and A2 [D2]. When this root string is next conceived contrapuntally, it is the component third motion that wins out. The "odd" parsing occurs as the brackets shift: D-B, C-A, etc. and G-B, A-C, etc. The appropriate contrapuntal sequence labels are D2 [D3] and A2 [A3].[28]

When the time is right, we will run the harmonic and contrapuntal processes side by side to obtain pairs of ordered sequences that function differently despite their identical source intervallic content. To acquaint ourselves with the familial relations possible among ordered sequences, though, we first consider the

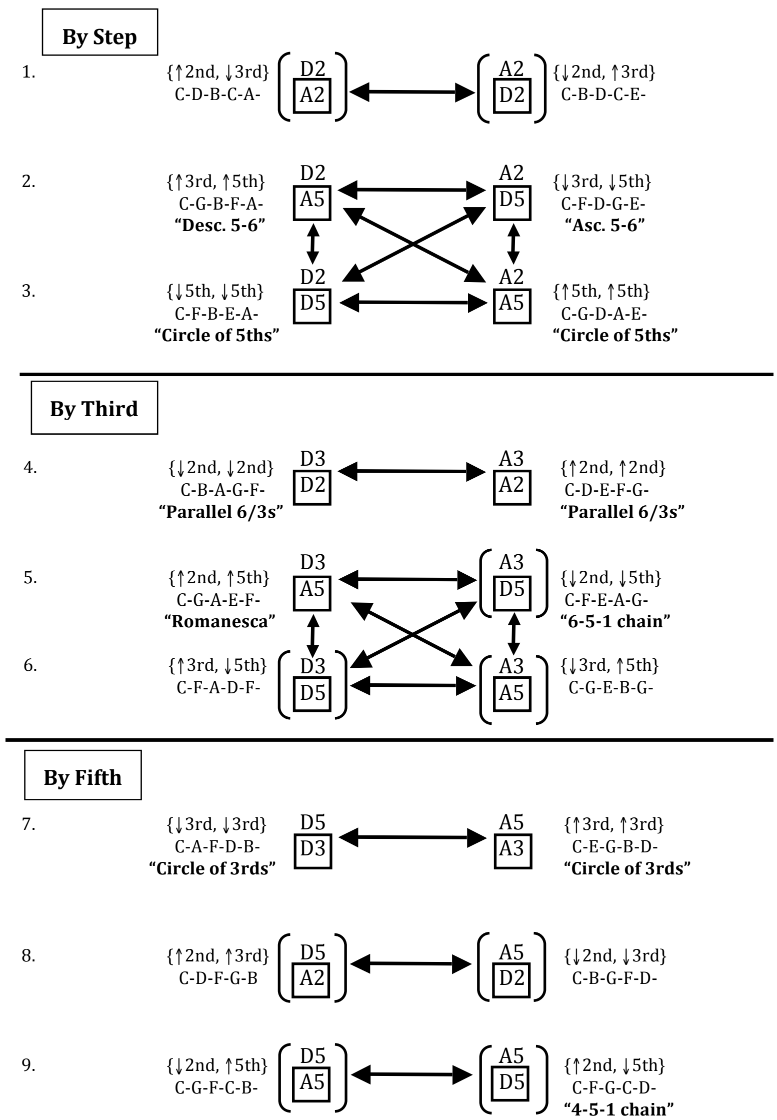
[28] I acknowledge that root pc-interval labels enfold a degree of cognitive dissonance in the present setting, since traditionally, contrapuntal motions are represented transformationally with figured bass symbols that disregard root content. This discomfort is overcome by asserting that the full function of model content is signified by the express character assigned by the theorist – that is: "harmonic" versus "contrapuntal" – and not by reading into the root descriptor given in brackets.

harmonic and contrapuntal shadings in isolation. The results of running the 18 unordered sequences through the harmonic model-parsing process are shown in Example 15. Sequence types that are rare in the literature have been placed in parentheses. In contrast to the procedure of Part II, the 18 parent unordered sequences are grouped by transposition level, by step, third, and fifth. The advantage of this arrangement is twofold. First, intervallic affinities will occur only among sequences in the same transpositional family. Second, the by-transposition organizational scheme provides a common framework for aligning the harmonic and contrapuntal charts in this and the upcoming example.

We begin our discussion of Example 15 by scanning the entries for patterns and anomalies. The sequences in Row 7 are the only ones to exhibit intervallic thirds in their models (see boxes). They appear in this chart only by virtue of the processing algorithm, which in handling class [3rd, 3rd] yields third motion as the model, even though it has no measurable harmonic quality. All other harmonically-functioning sequences have 5th-based and 2nd-based models. The strongly harmonic, fifths-based sequences appear in Rows 2, 3, 5, 6, and 9. Five out of these ten are in common use. In contrast, in the abstract there are only six weakly harmonic, 2nds-based sequences (Rows 1, 4, and 8), and four of them are virtually unheard of in the literature, those in Rows 1 and 8. This leaves only the two parallel first-inversion triad successions in Row 4 as auxiliary members of the harmonic ordered sequence group.

Further details about relationships among harmonic ordered sequences may be gleaned from their arrangement within each transpositional category. East-west oppositions in Example 15 indicate complete directional inversion. For example, the D2 [D5] sequence in Row 3 has both of its "D" terms reversed in its circle-of-fifths counterpart, A2 [A5]. Similar arrows appear between D2 [A5] and A2 [D5] in Row 2, indicating the essential sameness of descending and ascending 5-6 techniques. Special attention should be directed to the least obvious of these lateral relationships, that in

Example 15. Chart of the 18 ordered harmonic sequences. Parent unordered sequences are given at far left and far right along with sample root string and nickname, where relevant.



Row 5 linking D3 [A5] and A3 [D5]. The latter of these is a 6-5-1 chain, so named for a common chromatic modulatory scheme in which each local tonic arrival is reinterpreted as scale-degree 6 of the next three-chord iteration.[29] It is likely that most theorists would not immediately associate the Romanesca succession with 6-5-1 chain technique; however, the chart indicates their close kinship.

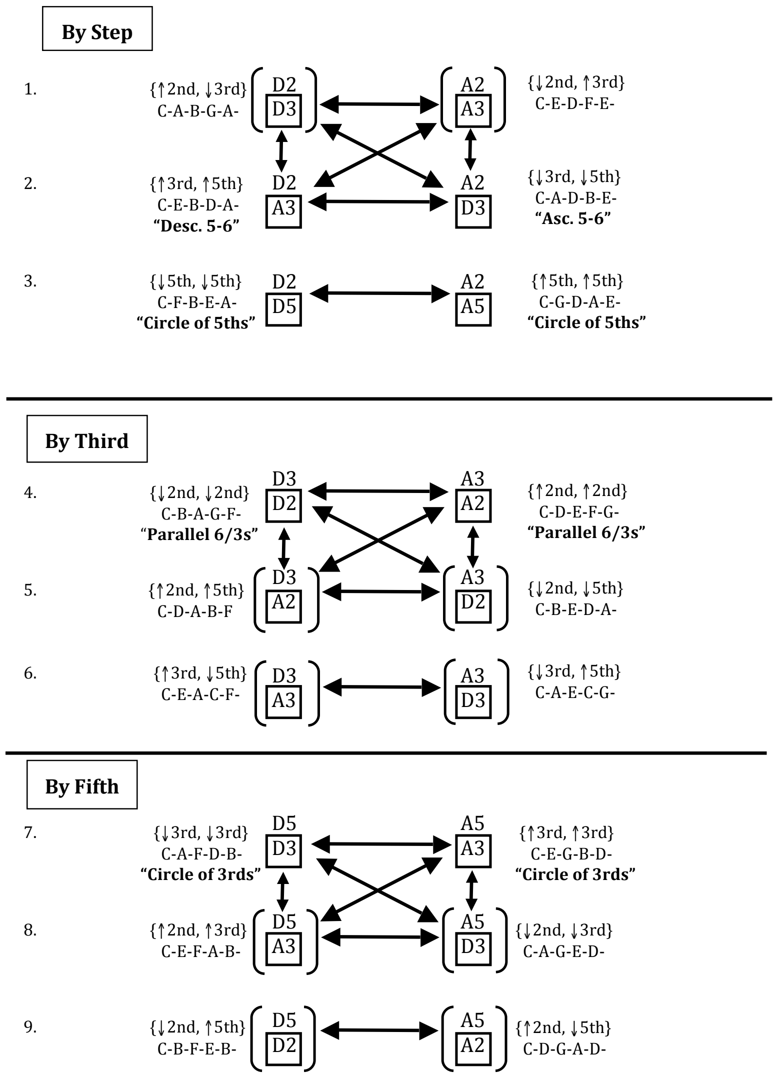
The other affinity tracked in the example is partial directional inversion. This occurs when one component of the sequence label is reversed, either the internal model interval (see north-south connectors) or the external transposition interval (diagonals). Where partial inversions are present, they create tight, 4-node networks. In the "By Step" group near the top, for instance, the four commonplace harmonic sequences in lines 2 and 3 are built of the same materials, root seconds and root fifths. This indicates close similarities between all 5-6 processes and all circle-of-fifths successions. In the "By Third" group, the same relations hold for the four member sequences appearing in Rows 5 and 6. This subgroup is bound together by reliance solely on thirds and fifths motion. Intriguingly, one may note that the entire "By Fifth" class has no such arrangement of closely related sequences. The maximum diversity of root types in this category—it contains models built of 2nds, 3rds, and 5ths—marks it as essentially unique.

The results of parsing the full set of unordered sequences to maximize their contrapuntal content are shown in Example 16. Many, though not all, of the root pc-interval labels are altered from before, as harmonic model content is replaced with the alternately parsed contrapuntal content. The basic format of the chart is unchanged, however, such that each of the 18 spots on this grid corresponds precisely with that of the previous example.

---

[29] When extended, the 6-5-1 pattern migrates through all 24 major and minor keys without repetition. The local arrival points, alternating major and minor triads, manifest as a neo-Riemannian RL cycle. A detailed account of this binary cycle and the mathematical-group space defining it may be found in Cohn 1997 (36-37).

Example 16. Chart of the 18 ordered contrapuntal sequences. Parent unordered sequences are given at far left and right with sample root string and nickname, where relevant

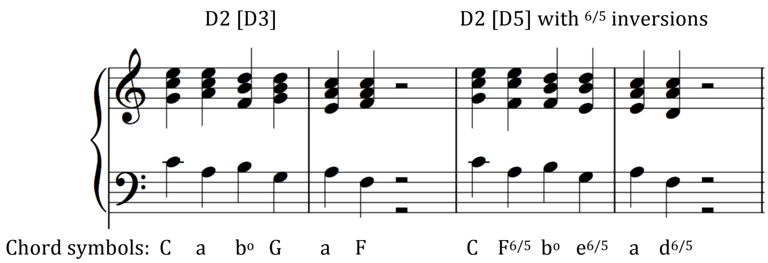


The double-headed arrows track the same relationships as before: total inversion of model and transposition intervals (horizontal arrows) and their partial inversions (all other arrows). Due to asymmetries in functional characteristics, the chart here does not mirror the harmonic chart. For one, Example 16 exhibits four-node subnetworks in all three transpositional zones (instead of just the first two). In the "By Step" area near the top, this arrangement appears among the four ordered sequences with thirds-based models; the circles of fifths do not participate. In the "By Third" area, it is formed among the four weakly contrapuntal sequences with seconds-based models; the two circles of thirds do not participate. In the last, "By Fifth" area, it once more occurs among third-based models.

Another way that this example diverges from the previous one concerns the number of member sequences deemed more or less likely to appear in music. Ten of 18 sequences are enclosed in parentheses here, as opposed to nine in the harmonic chart. The discrepancy involves unordered sequence {↑2nd, ↑5th} in Column 1, Row 5. Parsed harmonically, the class yields the familiar Romanesca, but parsed contrapuntally, it produces an awkward and unlikely D3 [A2] succession.

At first the number of harmonic and contrapuntal sequences in common use might seem fairly evenly split – nine to eight – but these initial numbers are misleading. Of the ten contrapuntal ordered sequences deemed *un*likely to occur, there is just one that can be "redeemed" by its close resemblance to a more familiar sequence. It is the D2 [D3] appearing in Column 1, Row 1. As Example 17 illustrates, this pattern closely corresponds to a D2 [D5] sequence utilizing inversion: it differs only by a single note in an inner voice. Allowing for the provisional inclusion of the D2 [D3], the number of contrapuntal ordered sequences likely to occur in the literature increases from eight to nine.

Example 17. Illustration of near-identity between contrapuntally-shaded sequence D2 [D3] and harmonically-shaded D2 [D5]



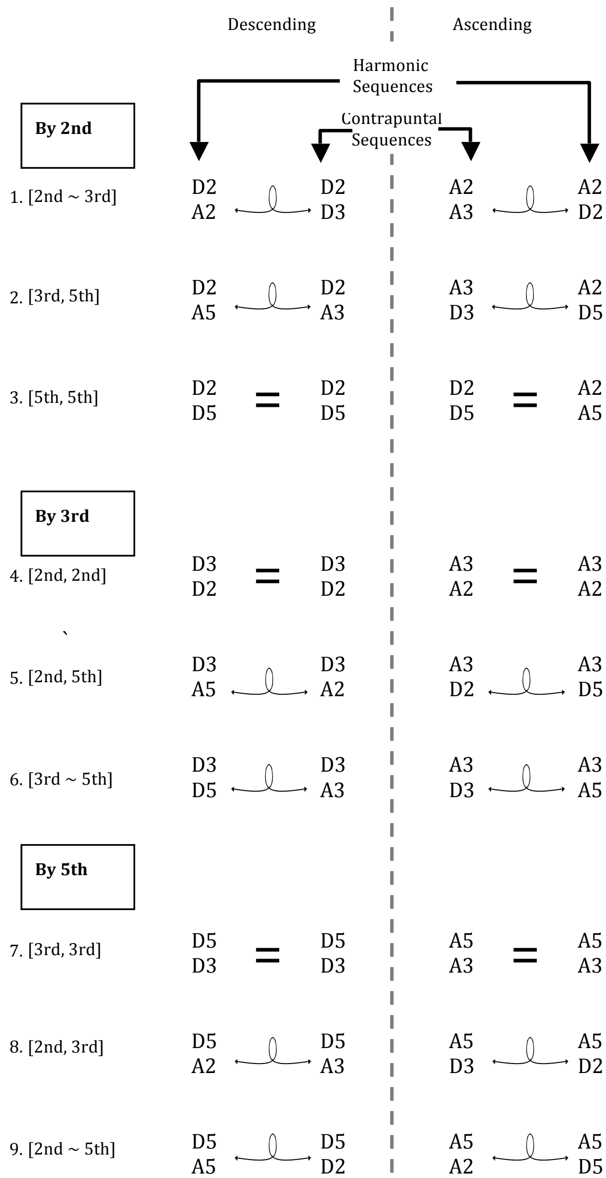
In contrast, three of the sequences appearing in parentheses in the harmonic chart —signaling their unlikely appearance, at least in diatonic form—can escape them via aid of chromaticism. It was noted earlier that harmonic sequences A3 [D5] and A5 [D5] are frequently used to modulate in the form of 6-5-1 and 4-5-1 chains. To these we may add the D3 [D5] succession from Column 1, Row 6. When chromaticized, this pattern produces a convincing descending third sequence that employs applied dominants and altered diatonic triads:

C – E – a – C – F – A – d – F – B♭

Incorporating this new information skews the balance further. There are now twelve types of harmonic sequence likely to appear in tonal music as opposed to nine types of contrapuntal sequence. Of course, this ratio can offer no insight into the question of which sequence shading, harmonic or contrapuntal, is more prevalent. It supports only the claim that harmonic sequence as a category possesses more internal diversity than the contrapuntal category.

The next stage towards classification involves collating the findings observed independently for harmonic and contrapuntal groups. Example 18 presents a preliminary organization, which allows us to observe interrelationships among all 30 two-chord sequences.

Example 18. Preliminary chart collating ordered harmonic and contrapuntal sequences. Parent sequence classes are shown at left. (Note: lower label boxes have been omitted)

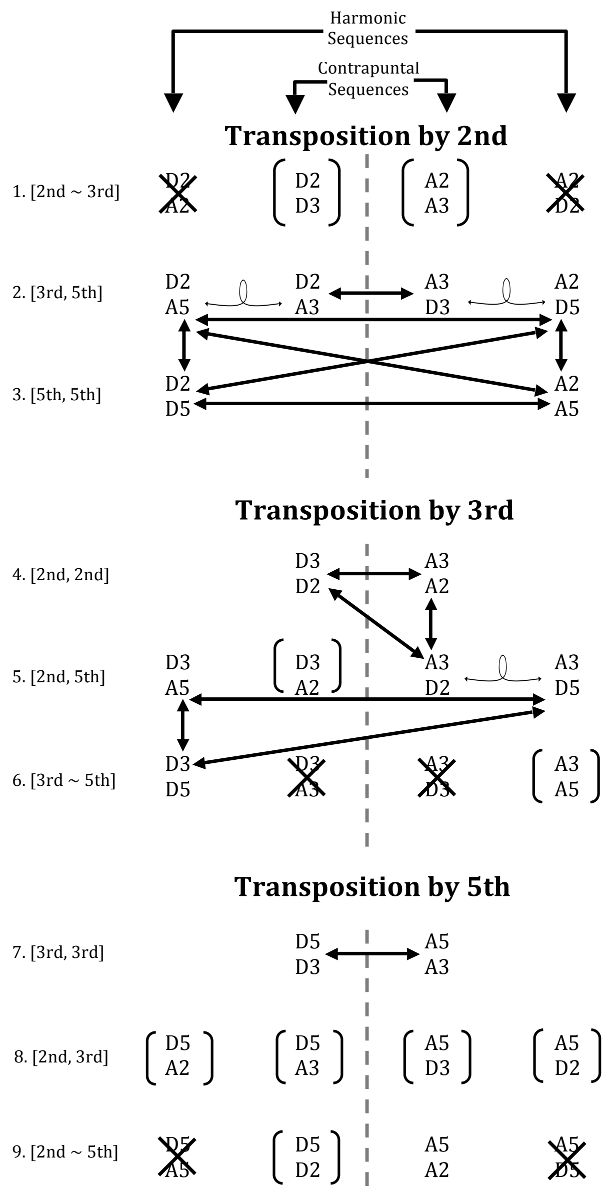


The four entries in each row reflect the four possible parsings of each sequence class in descending versus ascending transposition and harmonic versus contrapuntal model content. The former dichotomy needs little explanation: as before, descending sequences appear in the left half of the chart and ascending ones in the right.

As regards functional shading, the designations appearing in the 36 nodes of the diagram assess concretely what it means to parse a sequence one way or the other. Duplications, indicated by equal signs, are present in Rows 3, 4, and 7 due to there being only a single component interval class. There is only one way that [5th, 5th] can be parsed: harmonically. The same holds for classes [2nd, 2nd] and [3rd, 3rd]. The duplicate output in these rows is responsible for the 36 nodes reducing to 30 ordered sequences. The looping arrows are used to track a looser kind of equivalence that spans the harmonic/contrapuntal boundary. As the transpositional intervals hold constant, the model content shifts to reflect the two alternate parsings of the same interval root pattern (e.g., D2 [A5] and D2 [A3] in Row 2).

The preliminary chart in Example 18 is elaborated in Example 19 to track all the relationship types prioritized in this study. This requires not only the addition of relational arrows, but, more critically, the deletion of anomalous nodes. In line with observations concerning duplicate output in Rows 3, 4, and 7, two labels in each have been erased. In Row 3, the interior nodes have been removed, leaving the D2 [D5] and A2 [A5] to function as unequivocal harmonic sequences. The reverse occurs in Row 7, where the two circles of thirds are retained only in their contrapuntal capacity. In Row 4, the circles of seconds, the interior nodes have been eliminated to reflect the weakly harmonic character of this sequence class.

Example 19. The composite network for two-chord sequences



The rationale motivating the next set of deletions is best understood in the context of the relational networks. The horizontal, vertical, and diagonal arrows indicate the same full and partial intervallic inversions as before. By virtue of examining both parsings of each ordered sequence, however, the network in Example 19 provides new information concerning sequence families. A new outlier group emerges that includes the ordered sequences found in Row 1, Positions 1 and 4; Row 6, Positions 2 and 3; and Row 9, Positions 1 and 4. These six sequences, all rare in the literature, are unique for having no full or partial inversional partner. Each, in essence, acts as its own inversional partner, since its transpositional and model motions directly oppose each other. Due to this arrangement, once any of these sequences is initiated, it will return to its starting triadic root in four chords.

Theoretically, there is no rule that prohibits two-chord sequences from being structured in this way. We might thus be tempted to relegate them to non-viable status on the basis of their short, four-chord duration. Further consideration reveals, though, that it is not the length of these patterns, per se, that explains their intractability. After all, the Romanesca returns to its original root as early as its sixth chord.[30] The more likely cause of their problematic status is that two-chord patterns that exhibit rapid root return universally resist sequential interpretation, particularly when appearing in purely diatonic settings. This tendency may be observed first for the D3 [A3] and A3 [D3] sequences shown in Example 20. These were earlier designated as highly contrapuntal entities, and for that reason have been set here with the smoothest possible voice leading. In both cases, the chord successions that result are nearly indistinguishable from the composing-out of a single sonority. In Example 20a, a $C^{5/3}$ chord rotates to a C^6 chord

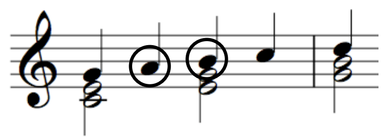
[30] Its shift of location inside the pattern is what keeps the return of the initial root sonority from sounding redundant. What had been an initiating, strong-beat chord is now heard as a weak-beat chord, or as some would have it, a "voice-leading" chord. The full theoretical and conceptual ramifications of this aspect of sequences are explored by Kochavi 2008 and Clough 2008.

as two non-chord tones embellish. The effect is the same in Example 20b, in which all aspects of the progression are reversed.[31]

Example 20. Perception of non-chord tones weakens sensation of chordal sequence

a.

A3 [D3]

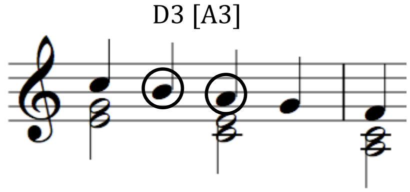


Strongly-perceived chords: C C^6 G

Weakly perceived chords: a^6 e

b.

D3 [A3]



Strongly-perceived chords: C^6 C F

Weakly perceived chords: e a^6

[31] It is fair to infer from this argument that rapid root-returning sequences are far more common in tonal music than commonly acknowledged. As the present aim is to fit the classification system with standard analytic practice, I stand by my assessment of such trivial sequences as "problematic." Future theorists wishing to expand the purview of chordal sequence, perhaps on the way toward recasting this phenomenon as an *a priori* impulse in tonality, surely will assess these particular entities differently.

This point is corroborated by the behavior of another four-chord member of this group, ordered sequence A5 [D5]. A comparative account of two prospective A5 [D5] sequences will indicate how difficult it is for composers to overcome listeners' instinctive functional-harmonic stance towards it. Example 21a is the only passage from the literature that Ricci cites as exemplifying A5 [D5].[32] The chromaticism in the bass voice has no bearing on the root succession, so it is fully correct to apply this label. Nevertheless, the prominent applied leading tones, A♯ and E♯, detract strongly from the sense of sequence. The passage does not sound like a circular motion from an F♯ chord back to F♯; it sounds like a directed progression *in* the key of F♯.

The progression shown in Example 21b from a Brahms Capriccio is a more convincing example of an A5 [D5] sequence. The complete four-chord sequence is presented first in G♯ minor and then in E Lydian; it is fully diatonic in both cases. The absence of chromaticism already makes it somewhat resistant to functional and/or Roman numeral analysis. Further contributing to the sense of sequence are the wide leaps in the bass voice. The registral disjunction is a clever artifice: Brahms realizes each pitch-class second as a compound seventh to mimic the bassline behavior of a conventional harmonic sequence.[33]

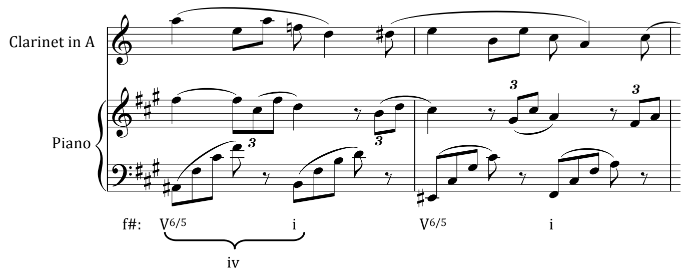
---

[32] Ricci 2002, 21.

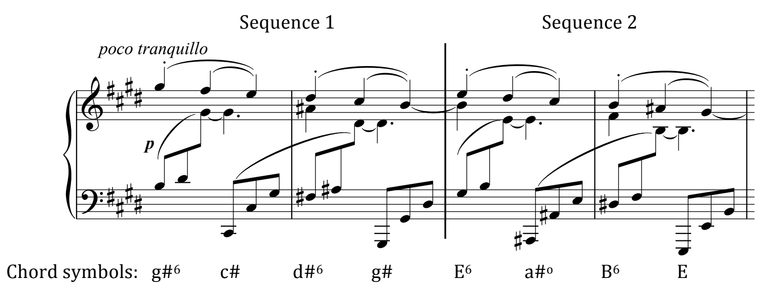
[33] Another anecdotal example comes from mm. 1-7 of the minuet movement of Schubert's Sixth Quartet in D, D. 74. It is possible to analyze this expository phrase as a sequential progression, viewing the D-G-A-D root succession as A5 [D5]. Each year that I assign this excerpt to undergraduates, a small number, taking their cue from the two-bar melodic patterning, offer precisely that analysis. They are vastly outnumbered, however, by classmates who view it as a simple I-IV-V-I progression.

Example 21. Two prospective A5 [D5] sequences.

a) Schumann, Op. 73, no. 3, mm. 11-12: Chromaticism clouds A5 [D5]'s identity as a chordal sequence. Roman numeral analysis supplied by the author.



b) Brahms, Op. 76, No. 5, mm. 53-57: two clearer instances of A5 [D5] created with diatonic triads. Pronounced bass leaps trigger model-copy sensation.



The extraordinary construction of this D5 [A5] is the exception that proves the rule. In light of the evidence assembled on the previous four pages, the six self-inverting nodes in Example 20 are deemed extraneous to common-practice sequence treatment and are crossed out.

The last set of deletions in Example 19, signaled by placement in parentheses, again indicate rarity in musical contexts. Three such sequences are scattered among Rows 5, 6 and 9, including D3 [A2], the reverse Romanesca. The bulk of deletions are found in Rows 1 and 8, the zones controlled by the two parent classes that were earlier seen as lacking strong characteristics in all domains. The fact that these six sequences include both 2nd- and 3rd-based models masks a larger trend, which is that 2nd-based sequences are simply rare in all contexts outside of parallel six-three triadic motion. The closest thing to an exception to this generalization is sequence A5 [A2] (Row 9). Wholly obscure in its diatonic form, it routinely manifests only chromatically, as a 4-5-1 chain.

Up until now, we have concentrated on three criteria, each indicated by its own notational strike-through convention, for sequential formulas to be disfavored by composers. We will next concern ourselves with examining the number and kinds of relationships manifesting among the remaining two-level patterns.

First at the highest level, a clear ranking among the three large sequence families is discernable. Both the "By 2nd" and "By 3rd" families contain six viable sequence types; however, the organization in the former group is more elegant. The box network encompassing Rows 2 and 3 ties all of the sequences together. Members related by diagonal arrows are conceptually and phenomenally similar in that they nudge a common model in opposing directions, while those related by vertical arrows are experienced as authentic versus plagal models moving the same direction in pitch space. This arrangement is further noteworthy for furnishing the largest number of viable connections within any transpositional family (7) and for bundling the six harmonic/contrapuntal sequences most common in tonal practice.

The active nodes in the "By 3rd" family are arranged more haphazardly. A novel feature here involves the two successions sequestered at the top. These parallel 6/3 successions have a strong association with each other, but connect only weakly to the sequences below. Unable to join to the reverse-Romanesca D3 [A2], their only potential link is with A3 [D2]. But even this paired diagonal/vertical connection is less robust than it appears. Like the A5 [A2] discussed above, the A3 [D2] is only likely to appear chromaticized as a 6-5-1 chain (or in other words, with artificial harmonic boosting). The A3 [D2], moreover, lacks a literal relation to any remaining sequences in this family. It is only by virtue of its equivalence with A3 [D5] at the far right of Row 5 that the network remains continuous. In contrast with the elegant, rectangle network formed in the lower two rows of the "By 2nds" family, the analogous region here is an empty triangle. And it is not only that the relationships graphed in this "By 3rds" region are looser than those seen in Rows 1-3. There are less of them as well: only six double-headed arrows appear instead of seven.

Comparatively little needs to be said about the minimal, fractured network of the "By 5ths" family. Only three viable sequences populate it, and only one inversional relationship can be posited. As a result, this area can be mined only for trivial truths, such as 1) that circles of thirds, when heard as two-chord patterns, produce motion by fifth and 2) that only one meaningful "by 5th" sequence is ever used with any regularity, the chromatic 4-5-1 modulatory chain. A third point could be added. Continuing the trend initiated by the two zones above, it becomes increasingly clear that a network's degree of disorganization directly corresponds to the level of disfavor for its members in composition. Some may here object that this is an arbitrary result precipitated by placing all entries in Row 8 in parentheses. In response, I would argue that it is more an issue of style and repertory, as it is the music of the common practice that is under consideration here. In future cases where it is evident that a particular composer or well-defined practice values sequences with maximally melodic (i.e., 2nd-based) models, Example 19 could be redrafted with the parentheses redistributed as needed or eliminated.

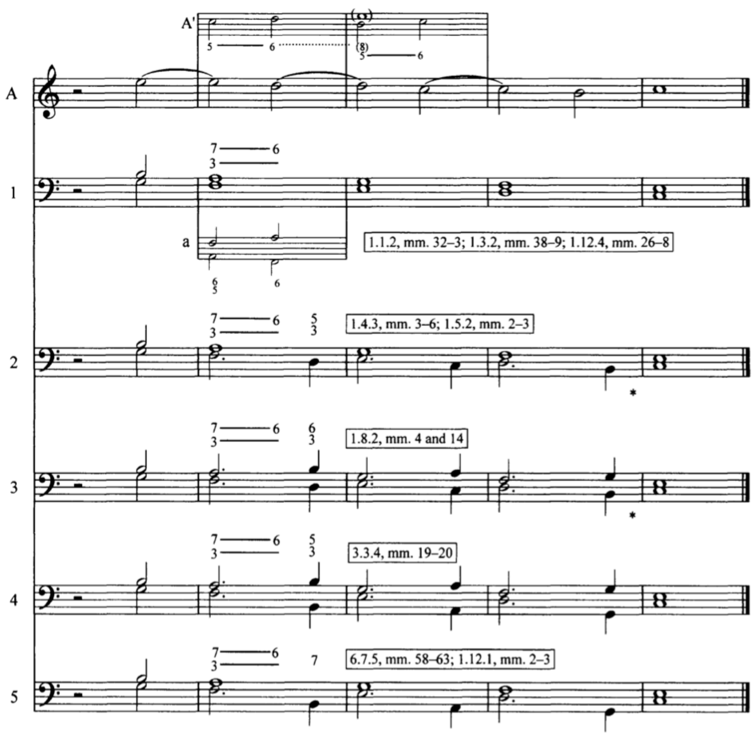
IV. Further Considerations

This closing section, dedicated to placing the findings from Parts II and III in context, will begin by comparing the present classification methods to those used in Harrison 2003 and Ricci 2002. Harrison's approach to sequence is historical. A detailed survey of the sequences appearing in instrumental works of Corelli and his contemporaries provides the basis for an evolutionary taxonomy. Harrison's central premise is that the oldest sequential passages in common use were contrapuntal in nature, rooted in suspension chains such as 7-6 and 2-3. These two-voice frameworks sensibly serve as the basis of his sequence classes, which incorporate harmonically-shaded phenomena as variants.

Example 22 (spanning two pages) shows two related classes of descending 7-6 sequence types, A and B. Harrison's complete set of seven classes, A-G, is summarized in Example 23.

Example 22. Two closely-related sequence classes from Harrison[34]

a) Class A sequence types (descending): 7-6 suspension chain in outer voices, inessential seventh chords



[34] Harrison 2003, 244-47.

Example 22. Two closely-related sequence classes from Harrison, continued.[35]

b) Class B sequence types (descending): 7-6 suspension chain in upper voices, essential seventh chords with bass in descending fifths

One benefit of his approach is that it offers a clear way to coordinate sequences' dual impulses: in this case, contrapuntal concerns always trump harmonic ones. Another is efficiency, as many sequence forms can be grouped by a shared trait. The common feature of the five Class A sequences is the 7-6 contrapuntal backbone appearing in the outer voices; in Class B, the chain always involves at least one inner voice. This broad grouping strategy allows one to observe genetic correspondences among seemingly unlike formations; note, for example, the near identity of parallel six-three chords and the circle of fifths progression in Example 22, Lines A1 and A4.[36]

---

[35] Harrison 2003, 244-47.

[36] In my Example 19, this relation is highly obscured. It emerges only when D2 [D5] is read in an accelerated time frame; i.e., under M2 extraction of every other element.

Example 23. Summary of the seven sequence classes from Harrison[37]

Direction	Type	Basic suspension	Characteristic feature
Descending	A	7–6	Inessential seventh-chords
	B	7–6	Essential seventh-chords
	C	7–6	Double-length unit
	D	2–3	Bass suspension
Ascending	E	2–3	Braid
	F	2–3	Alternating braid
	G	5–6	Consonant syncope

Harrison's logic and exhaustive score evidence leave readers little reason to doubt his thesis. As revealing as his summary classification is, though, it remains impractical for several reasons. For one, it treats vertical sonorities inconsistently, allowing some to be recognized as chords and others not. For another, it is in one important regard incomplete. Though Harrison appropriately investigates the origins of some longer 3- and 4-sonority patterns, some theoretically important two-chord ones are overlooked. Here we must bear in mind that his classification is not, nor was it ever meant to be, a comprehensive theory of all sequence types. In concentrating on the diatonic sequences that Corelli explicitly developed, it is mute on the subject of sequences such as D3 [D5] and A2 [D5] that lack historically contrapuntal roots.

An alternate classification system is proposed in Ricci 2002. Wholly relegated to the second half of that paper, it oddly depends little on the "natural" diatonic set-theory groupings (cf. Example 3) that were so carefully laid out in the first half. His method instead groups sequence classes according to their voice-leading parsimony. The main metric is voice-leading distance (VLD), measured in terms of the fewest diatonic steps necessary to transform one complete triad in close position into another. The six root motions are scored in terms of their VLD in various settings. His noble impulse to account for the many possibilities of

[37] Harrison 2003, 254.

parallel versus non-parallel motion, revoicing, and voice overlaps results in a wide range of VLD values that go even beyond 12. Ricci's response to this conundrum is to establish broad voice-leading categories, or "genera," wherein sequences group by VLD ranges as shown in Example 24a. The ranges are sufficiently expansive so that only a single Sequence Class appears in Genus I, six in Genus II, and two in Genus III (Example 24b).

A comparison of Example 24b with the final sequence classification chart of Example 19 reveals the shortcomings of Ricci's approach. Consider first the four tightly interrelated sequences in Example 19, Row 2 deriving from parent class [3rd, 5th]. The problem is not that they are assigned elite status in Ricci's Genus 1, but that they are cut off from their close relatives in sequence class [5th, 5th]. Ricci's chart similarly cordons off the circles of seconds and thirds. This contrasts with my chart, which closely links Sequences D3 [D2] and A3 [A2] (Row 4) with the well-known A3 [D2] pattern. To note a final drawback, Ricci's taxonomy is impractical for including too many entities in Genus 2. Under the criteria of voice-leading, there is no way to distinguish among these six sequence classes and the 22 ordered sequences they produce.

Ricci's decision to classify sequences by voice leading is not in any way misguided. His sensitive treatment is invaluable for describing sequences in pitch space, where the same root-identified chords are likely to occur in different voice configurations. This condition cannot be modeled in the generic pitch-class space in which I am working. Yet speaking practically, the price of Ricci's fastidiousness is too high. The genera classifications, valid as they are, are unwieldy and have not been widely adopted.

Example 24. Sequence classification in Ricci 2002.

a) Criteria for the genera and species of organization based on voice-leading distance (VLD). Genus 1 is defined to contain VLDs 1-3, Genus 2 VLDs 4-6, and Genus 3 VLDs >7.

Genus	Species	
	a. non-parallel	b. parallel
1 (parsimonious)	*VLDs* 1, 2	*VLDs* 1, 2, 3
2	*VLDs* 1, 2, 4, 5	*VLDs* 1, 2, 3, 4, 5, 6
3	*VLDs* 1, 2, 4, 5, 8, 10, 11 . . .	*VLDs* 1, 2, 3, 4, 5, 6, >7

b. Sequence class membership by genus (adapted from Ricci 2002, 19)

Genus	Species	
	a. non-parallel	b. parallel
1 (parsimonious)	[3rd, 5th]	---
2	[2nd, 5th] [2nd~5th] [2nd~3rd] [3rd~5th] [5th, 5th]	[2nd, 3rd]
3 (voice overlaps)	[2nd, 2nd], [3rd, 3rd]	---

Ricci's work has obviously served as a foundation for this study, particularly with regard to its pioneering use of sequence classes. My decision to diverge from him is born out of the desire to make chordal sequence classification more intuitive and transparent. This has required a number of simplifications. This essay has considered only two-chord, purely triadic sequences. All distance calculations have been carried out in pitch-class space, rendering issues of inversion and bass voice contour irrelevant. Readers should note that this solution does not in any way preclude the possibility of re-introducing this information at a later stage of the analytical process. For example, it is easy to imagine a more refined functional classification scheme in which one starts out with a basic, root pc-interval label that carries only neutral significance. It could then be nudged toward the harmonic or contrapuntal ends of the functional spectrum depending on how the pattern at hand behaves with regard to parsing, chromaticism, and disposition the bass voice.

Full consideration of the complexities stemming from the incorporation of any further domains lies beyond the scope of an introductory essay such as this. The classification system here, therefore, has been rendered in a preliminary state to maximize its utility to future theorists. The ranking systems and resultant network graphs in Examples 11 and 19 may thus be regarded as a framework on which further findings may be grafted. Even so, it is sufficiently developed in this early form to provide some immediately tangible benefits.

Many of the projected applications of this theory are pedagogical. For the first time, all 30 ordered two-chord sequences are organized in a manner that clarifies their structure, interrelated identities, and functions. The chart in Example 19 will be useful to students who wish to investigate the full set of possibilities for two-chord constructions in terms of what is possible, what is musically likely, and what theoretical factors come to bear when negotiating the space between these conceptual realms. A further benefit to classroom instruction involves standardizing the labeling system for sequences. That endeavor requires a long overdue untangling of sequence function from nomenclature such that, in the future, the former need not always encode the latter. I have suggested a number of paths towards that goal, beginning with the basic step of rebranding these patterned phenomena as "chordal" or "triadic," but not necessarily "harmonic," sequences. Another involves running all diatonic sequence classes through both "harmonic" and "contrapuntal" filters as in Part III. This is done not to privilege one musical function over the other, but to evacuate the distinction between them; the looping equivalence arrows, for example, readily transform any entity parsed in one functional shading into its alternate form.

Whether or not a unified, comprehensive system of sequence nomenclature comes to pass, the arguments advanced here should be sufficient to reopen the discussion about chordal sequence behavior, serving as a potent reminder that such entities in all forms have inalienable contrapuntal, harmonic, and other still-underappreciated characteristics. At this remarkable point in time, in which musicians' interest in both the mathematical properties and analytical applications of chordal sequences has been rekindled, it is critical that we establish an objective means for describing them that is not rooted in habit or dogma. Even in cases in which theorists remain predisposed to "read" sequences in a particular fashion, it is critical that they be able to articulate their reasons for doing so.

Potential theoretical applications of this study derive from its aforementioned flexibility in that nearly all technical aspects of the method may be subjected to revision. A cosmetic shift might involve altering the 0-2 values advocated in Part II for ranking inherent harmonic, contrapuntal, and melodic character. New values could be substituted based on alternate theoretical principles, the outcomes of cognition experiments, preference, or some combination of all of these. Yet more intriguing is the idea of incorporating more radical alterations to the method; for example, privileging other chord-root motion characteristics, such as melodic character and contextual operations, over the classic "harmonic" and "contrapuntal" traits emphasized here.

A more dramatic shift – one that I hope can be explored in future research – involves redefining familial relationships among sequences and sequence classes. The only relationships tracked above were defined in terms of full and partial root-intervallic reversal. Though logical and audible, these equivalence relations are also to a certain extent trivial. A more important fact to bear in mind is that they are contextual. Above, the nine sequence classes were ordered according to their higher-order transposition scheme: this was done to ensure consistency with the current, prevailing "model-copy" view of sequences. Other equivalences are possible, however. An alternate system of organization might privilege model content, allowing for unordered sequence pairs such as {2nd, 5th} and {2nd ~ 5th} to be grouped within a functional class despite their differing transposition indices. Future work on this topic will require that other measures of similarity be theorized and coordinated.

For centuries, a unified system of sequence classification has proved elusive due to the inherently dichotomous nature of these patterns. It would be complicated enough to theorize the mathematics of nested transposition, where a surface chord string is viewed as a pattern of patterns. The perennial possibility of odd-versus-even parsing elevates the complexity by introducing a condition in which viewing a pattern from one perspective necessarily obscures the other. This is most familiarly experienced in terms of sequences' inherent harmonic/contrapuntal duality. Yet as shown earlier, the same condition is precipitated by sequences being grouped according to their level of transposition with dissimilar chord content disregarded, or vice versa.

This procedural paradox is reminiscent of Heisenberg's uncertainty principle. The closer one gets to measuring one aspect of sequence behavior, the more a different aspect is actively obscured. Or perhaps a discipline-specific metaphor might better communicate this point. The impossibility of ever truly "resolving" the sequence question resonates with music theory's most ancient concern of reconciling pitches generated by co-prime intervals (most famously, multiples of 2 and 3). Some musical puzzles, it seems, were never meant to be solved. This truth should not discourage us, any more than it has the legions of investigators who cumulatively have dedicated many lifetimes to studying Pythagorean tuning. For our goal in theorizing about music is not a single, neat conclusion. The motivation is rather more subtle: to gain knowledge through inquiry into a complicated natural or human-made phenomenon. In seeking a topic with sufficient mathematical and perceptual complexity to sustain prolonged future inquiry, it should be clear from this and other exploratory essays that one could hardly imagine a better candidate than that of chordal sequence.

References

Allsop, Peter. 1992. *The Italian 'Trio' Sonata: From its Origins until Corelli.* Oxford: Clarendon Press.

Aldwell, Edward and Carl Schachter. 2003. *Harmony and Voice Leading,* 3rd ed. Belmont, CA: Schirmer/Thomson Learning.

Bass, Richard. 1996. "From Gretchen to Tristan: The Changing Role of Harmonic Sequences in the Nineteenth Century." *19th-Century Music* 19: 263-285.

Clough, John. 1979. "Aspects of Diatonic Sets." *Journal of Music Theory* 23/1: 45-61.

-----. 1994. "Diatonic Interval Cycles and Hierarchical Structure." *Perspectives of New Music* 32: 228-25.

-----. 2008. "Flip-Flop Circles and Their Groups." In *Music Theory and Mathematics: Chords, Collections, and Transformations*. Edited by Jack Douthett, Martha Hyde, and Charles J. Smith. Rochester: University of Rochester Press: 23-48.

Cohn, Richard. 1997. "Neo-Riemannian Operations, Parsimonious Trichords, and Their 'Tonnetz' Representations." *Journal of Music Theory* 41/1: 1-66.

Crocker, Richard L. 1962. "Discant, Counterpoint, and Harmony." *Journal of the American Musicological Society* 15/1: 1-21.

Damschroder, David. 2006. "Schubert, Chromaticism, and the Ascending 5-6 Sequence." *Journal of Music Theory* 50/2: 253-275.

Fétis, Francois-Joseph. [1840] 1994. *Esquisse de l'histoire de l'harmonie*. Translated and and edited by Mary Arlin. Stuyvesant, NY: Pendragon Press.

Forte, Allen and Steven Gilbert. 1982. *Introduction to Schenkerian Analysis*. New York: W.W. Norton.

Gjerdingen, Robert. 2007. *Music in the Galant Style*. New York: Oxford University Press.

Harrison, Daniel. 2003. "Rosalia, Aloysius, and Arcangelo: A Genealogy of the Sequence." *Journal of Music Theory* 47: 225-272.

Hill, John Walter. 2005. *Baroque Music: Music in Western Europe, 1580-1750*. Urbana-Champaign: University of Illinois Press.

Kochavi, Jonathan. 2008. "A Parsimony Metric for Diatonic Sequences." In *Music Theory and Mathematics: Chords, Collections, and Transformations*. Edited by Jack Douthett, Martha Hyde, and Charles J. Smith. Rochester: University of Rochester Press: 174-196.

Laitz, Steven. 2012. *The Complete Musician: An Integrated Approach to Tonal Theory, Analysis and Listening*, 3rd ed. New York: Oxford University Press.

Moreno, Jairo. 2000. "Challenging Views of Sequential Repetition: From Satzlehre to Melodielehre." *Journal of Music Theory* 44/1: 127-169.

Rameau, Jean Philippe. [1722] 1971. *Treatise on Harmony*. Translated by Philip Gossett. New York: Dover Publications.

Ricci, Adam. 2002. "A Classification Scheme for Harmonic Sequences." *Theory and Practice* 27:1-36.

Schenker, Heinrich. [1935] 1979. *Free Composition*. Translated and Edited by Ernst Oster. New York: Longman.

Tymoczko, Dmitri. 2008. "Scale Theory, Serial Theory, and Voice Leading." *Music Analysis* 27/1: 1-49.

-----. 2011. *A Geometry of Music*. New York: Oxford University Press.