The Tempo-Span GIS as a Measure of Continuity in Elliott Carter's *Eight Pieces for Four Timpani*

Yayoi Uno

Introduction

Elliott Carter stands as one of the leading American composers to have developed a rhythmic language of sophistication and precision in the second half of this century. In particular, his technique of metric modulation has allowed for an exceptional rhythmic fluidity in his music written since the 1950s: shifts in tempo and meter are integrated with surface rhythms in such a way that the transitions from one tempo to another sound practically seamless.[1]

One of the questions posed by Carter's rhythmic practices, nonetheless, has to do with the function of meter. While his music retains a time signature throughout, the notated meter does not necessarily define a perceivable pulse. For instance, consider the excerpt from his *String Quartet No. 1* (1951) provided in Example 1. Here the 4/4 meter defines the beat at ♩ = 120MM, but a listener tends to discern four distinct pulses moving at different speeds, namely 36MM, 96MM, 180MM and 120MM (as shown).[2] The pulses for the first and second violin

[1]Jonathan Bernard, "The Evolution of Elliott Carter's Rhythmic Practice," *Perspectives of New Music* 26 (1988): 164–201. Jonathan Bernard, in tracing the evolution in Carter's rhythmic practices, comments on Carter's preoccupation with successive temporal changes involving metric modulations in early works beginning with the *Cello Sonata* (1948): "Carter at the beginning of this evolutionary period [late 1940s to early 1950s] was more preoccupied with techniques of rhythmic *succession* than those of *simultaneity*.... Simultaneity emerges in an increasingly integral relationship to the substance of the works after 1948, as Carter's confidence in his handling of rhythmic-successive techniques grew." (p. 167) Bernard discusses how specific speeds become associated with specific pitch or intervallic entities in later works such as the *Second String Quartet* (1959) and the *Double Concerto* (1961).

[2]See David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987), pp. 60–86. Lewin discusses this excerpt

are articulated by recurring timespans that move slower than 120MM (as shown). Even if the cello line retains a quarter-note beat as a pulse, melodic contour and slurring cut across the barlines and fail to reinforce the accentual scheme of 4/4. It is assumed that the time signature is there primarily for notational convenience—to coordinate the four instruments through a common metric framework in order to show the rhythmic alignment and interrelationship of parts.

The impetus for the present analysis stems, indeed, from the necessity to reexamine the function of meter in Carter's music. David Schiff claims that the term 'metric modulation' is a misnomer, and prefers to use the term 'tempo modulation.'[3] More specifically, I argue that Carter's music (composed since the late 1940s) is non-metric because (1) the notated beat (associated with the notated meter) does not necessarily articulate the musical pulse, and (2) the cyclical regularity and accentual scheme of strong and weak beats—factors that define the function of meter—are largely missing. In order to assess the temporal continuity in Carter's music with greater precision, I have constructed an analytical method that quantifies the local changes in the speed and grouping of the pulse based on David Lewin's theory of generalized interval systems (abbreviated to GIS).[4] My analytical method, the *tempo-span* GIS, will be

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from Carter's *String Quartet No. 2* in reference to his formulation of a time-span GIS, as will be discussed.

[3]David Schiff, *The Music of Elliott Carter* (London: Eulenberg Books, 1983), p. 26. "Different speeds appear as others fade out, so that the overall tempo of the music fluctuates rapidly. The notational means for achieving this effect is known as 'metric modulation'; this term is slightly misleading, because the metre does not really change—since there is no metre. Carter now prefers 'tempo modulation'." In the context of the present analysis, I employ the original term because it has acquired certain currency in the literature having to do with Elliott Carter's music. I find *tempo* modulation to be an equally misleading designation in that the term *tempo* is fraught with conflicting musical connotations throughout history.

[4]Lewin, *Generalized Musical Intervals and Transformations.* My *tempo-span* GIS is an adaptation of Lewin's time-span GIS, as will be discussed; also see David Lewin, *Musical Form and Transformation: 4 Analytic Essays* (New Haven: Yale University Press, 1993).

Example 1. Elliott Carter, String Quartet No. 1 (1951): mm. 23–31.

applied to two movements, "Canaries" and "March," from Carter's *Eight Pieces for Four Timpani* (1950).[5]

Figure 1 provides working definitions for relevant terms for this study. Drawing from previous studies by Jonathan Kramer[6] and David Epstein,[7] I distinguish between *beat* and *pulse* on the basis of metric framework and types of accents. Beats are constituents of meter and its hierarchical framework, while pulses are undifferentiated timespans—foreground rhythmic entities marked off by contextual accents.[8] Following John Roeder, but

[5]Jonathan Bernard has informed me that the timpani works (1950) were conceived as study pieces for the *First String Quartet* (1951).

[6]See Jonathan D. Kramer, *Time in Music: New Meanings, New Temporalities, New Listening Strategies* (New York: Schirmer Books, 1988), p. 97. "Beats are timepoints...the meter of the music. Pulses, however, are flexible, and they are rhythmic. They are events in the music that occur at or near beats.... A pulse is literally heard, not intuited the way a beat is. Pulse is susceptible to rhythmic accent, while metric accents are applied to beats.... The two may or may not coincide, but they are conceptually—and *experientially*—distinct. A pulse is an event in the music, interpreted by a performer and directly heard by a listener. It occurs at a timepoint. A beat, on the other hand, is a timepoint rather than a duration in time." My definitions of *beat* and *pulse* differ from Kramer in asserting that they are *timespans* marked off by metric or contextual (not only rhythmic) accents.

[7]See David Epstein, *Shaping Time: Music, the Brain and Performance* (New York: Schirmer Books, 1995), p. 29. "The most significant aspects of *beat*, the primary level of meter, concern not only its durationally equal spacing, but the manner of accent.... *Pulse*, the primary durational level of rhythm, is the complementary unit to beat with regard to meter. While metric beat carries the "neutral" marked mode of accent just discussed, pulse enjoys a far broader spectrum of accentual articulations, the variety potentially as great as the character of musical works themselves."

[8]*Ibid.*, p. 43. "A number of writers have suggested that meter can only be inferred through the interaction of metrical hierarchies. The essence of this argument is that an undifferentiated sequence of beats, all of them equal in duration, durational spacing, intensity, and so on, will not be understood as meter until and unless the beats are further perceived in connection with the next higher metrical framework, that of the measure. The measure imposes upon the beats the more complex organization of pattern and boundary limits. From these bounded patterns we infer (or to them we ascribe) the further property of accent(s). This interaction continues among ever higher levels of metric hierarchy." Also see Fred Lerdahl and Ray Jackendoff, *A Generative*

Figure 1. Definitions of relevant terms.

beat: timespan that defines the accentual relationship of strong and weak beats in the context of a meter.

pulse: timespan that articulates a temporal ictus through contextual means, *e.g.*, rhythmic grouping, melodic contour, agogic, and/or dynamic emphasis.

pulse stream: a succession of pulses consisting of equal timespans (metrically undifferentiated).

subpulse: timespans that subdivide the pulse into equal units. Subpulses receive weaker articulation relative to a pulse.

speed: an absolute measure (definable by metronomic indication) of the frequency of a given pulse or a subpulse.

metric modulation: transference in speed between unequal rhythmic durations that may induce a change in the grouping and/or the speed of a pulse.

with a slightly different denotation, I define a *pulse stream* as a succession of pulses consisting of equal timespans; a minimum of two equal timespans is necessary to activate a pulse stream.[9]

Musical contexts dictate the articulation and grouping of pulses within a pulse stream. In Example 1, four pulse streams are established by the four instruments through the repetition of distinct timespans, marked off by contextual accents. While the

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Theory of Tonal Music (Cambridge: MIT Press, 1983); Maury Yeston, *The Stratification of Musical Rhythm* (New Haven: Yale University Press, 1976).

[9]John Roeder, "Interacting Pulse Streams in Schoenberg's Atonal Polyphony," *Music Theory Spectrum* 16/2 (1994): 231-249. My definition of a *pulse stream* is commensurate with Roeder's *pulse*: "A pulse is a series of successive, perceptibly equal timespans, marked off by accented timepoints..." (p. 234). Roeder allows for multiple pulse streams to run concurrently, while I work with a single pulse stream at a time that is liable to change from bar to bar. In Carter's "Canaries" and "March," however, there are passages where I have constructed two pulse streams that unfold in counterpoint with each other.

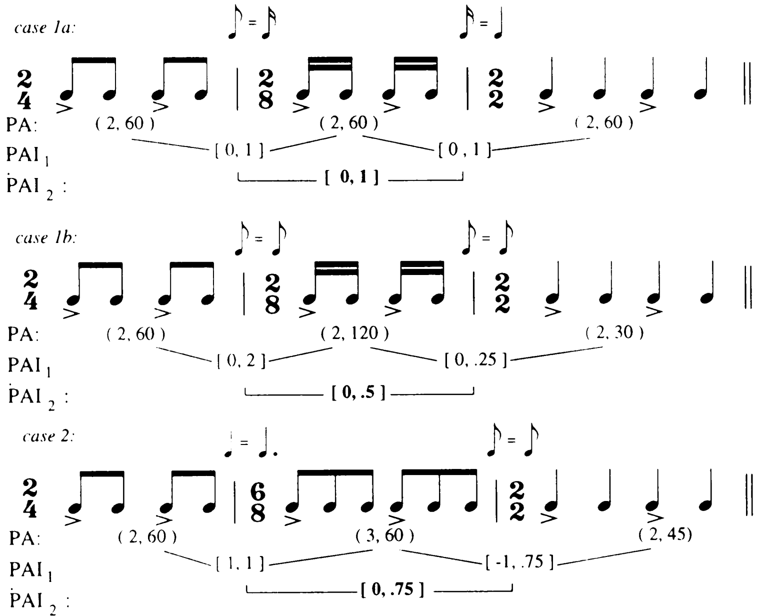
cello part provides a steady series of quarter notes (mm. 22–26), melodic contour and slurring generate contextual accents that override the metric accents associated with a 4/4 meter. We therefore tend to hear this line as a pulse stream rather than as a metric grouping in 4/4. The pulse stream in the viola line (mm. 25–29) is, however, ambivalent. One can hear the viola line as a pulse stream of ♪(3) = 180MM or, alternately, as a pulse stream of 𝅗𝅥 = 60MM with three *subpulses*. The repetition in melodic contour and frequent registral gaps accentuate the onset of each triplet so as to impart a metric accent associated with 3/2.

Throughout this analysis, Carter's term *speed* is used to define an absolute measure of the velocity of a given pulse or subpulse (definable by the metronomic marking).[10] For instance, the initial speed of the pulse stream for the cello is ♩ = 120MM. A tempo change, in the context of this paper, denotes a change in the speed of a pulse from one section to another, expressed as a ratio. In Example 1, where the pulse stream of the cello slows down from 120MM to 48MM, the tempo has slowed down by a 5:2 ratio.

Finally, metric modulation can be defined specifically as a transference in speed between unequal rhythmic values that may induce a change in the grouping and/or the speed of a pulse. Example 1 shows how metric modulations are used in two distinctive ways. The speed and grouping of the pulse remains unchanged across the barline between mm. 21 and 22, but the speed of the notated beat is changed (the cello's dotted eighths in m. 21 are of the same speed as the cello's quarter notes in m. 22) and the meter changes from 3/8 to 4/4. In the second metric modulation (mm. 30–31), the former speed of a half note is applied to a notated dotted half note at m. 31 (m. 30 in the viola); this initiates a change in the grouping of the pulse from triple to duple and a change in the speed of the pulse from 60MM to 90MM.

[10]See Else Stone and Kurt Stone (eds.), *The Writings of Elliott Carter: An American Composer Looks at Modern Music* (Bloomington: Indiana University Press, 1977).

Example 2. Hypothetical contexts for metric modulation.



Given these working definitions, how do we then measure the successive changes in the structure of pulse over time? Example 2 presents simple musical contexts to illustrate how a ***tempo-span*** GIS analysis measures the changes in the grouping and/or the speed of pulse. Cases 1a, 1b, and 2 involve changing meters, but only 1a and 2 (in which rhythmically unequal durations are equated) involve metric modulations. In the first two cases, rhythmic grouping and accents consistently articulate a duple subdivision of the pulse. The tri-level network of ordered intervals below the musical notes comprise a tempo-span GIS network. The ordered pairs in round brackets immediately below the notes denote the changes in ***pulse attributes*** (hereafter abbreviated PAs); they comprise an ordered pair of positive real numbers where the former denotes the grouping or subdivision of the pulse and the latter denotes the speed of the pulse. Notice how the PAs model the "perceived" changes in the pulse structure: the speed and grouping remain the same for case 1a, while the speed of the pulse changes from 60MM to 120MM, and then to 30MM in case 1b. Case 1a shows a simple illustration of how metric modulation can be used to facilitate the transition from one pulse stream to another by equalizing the speed of pulses or subpulses.

The ordered pairs in square brackets below the PAs are *pulse attribute intervals* (hereafter abbreviated to PAIs) containing the difference in the grouping of pulse and the ratio of the speed of pulse between two PAs. The PAI between two PAs (*a*, *b*) and (*c*, *d*) would be [*c–a*, *d*/*b*]. A "first-order PAI" (PAI_1) measures the intervals between consecutive PAs; a "second-order PAI" (PAI_2) measures PAs two apart. Every PAI_2 is distinguished from a PAI_1 by its boldface type. In case 1a, [0, 1] indicates that neither the grouping of the pulse nor the speed undergoes any change. In case 1b, the PAI_1s indicate that the tempo quickens to twice the speed then slows down by a factor of four; the net tempo reduction by a factor of 2 appears as ".5" in the PAI_2. In case 2, the PAI_2 [0, 0.75] indicates that the increase and decrease in grouping by 1 results in no change, while the tempo slows down from the start by 25 percent.[11]

Modeled after Lewin's time-span GIS (of which his definition is reproduced in Figure 2a), the mathematical formalization for my tempo-span GIS is provided in Figure 2b.[12] My tempo-span

[11]My calculation of the percentage increase or decrease between ratios is based on the following formula: given a ratio *x*:*y* in which $x<y$, then the percentage of acceleration from *x* to *y* equals $(y–x)/y$. For example, the ratio 3:4 denotes an increase of (4–3)/4 = 1/4 or 25% faster. A ratio of 2:3 denotes an increase of (3–2)/3 = 1/3 or 33% faster. If, however, $x>y$, then the percentage of deceleration equals $(x–y)/x$. For example, the ratio 2:1 denotes a decrease of (2–1)/2 = 1/2 or 50% slower. This is contrary to normal usage for a reason: typically, we might prefer to label a 2:3 acceleration as "becoming half again as fast," while describing the inverse of this acceleration, a 3:2 deceleration, as "becoming a third slower." Using my alternate method above, however, two such inversely-related tempo changes are given the same percentage quantity, and modified with the words "slower" or "faster."

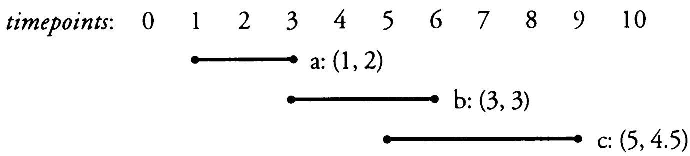
[12]Lewin, *Generalized Musical Intervals and Transformations*, pp. 60–61. Lewin contrasts this commutative GIS later with a non-commutative GIS that models the perception of time-span proportion between *int* values more

Figure 2a. Lewin's definition of a time-span GIS (from Generalized Musical Intervals and Transformations, *pp. 60–61).*

By a time-span, we will understand an ordered pair (a, x) where a is any real number and x is any positive real number. The pair of numbers is understood to model our sense of location and extension about a musical event that "begins at time a" and "extends x units of time." The family of all time-spans will be denoted TMSPS.

Take S = TMSPS. Take IVLS to be the direct-product group of the real-numbers-under-addition by the positive-reals-under-multiplication. Define the function int, from S × S into IVLS, by the formula $\text{int}((a, x), (b, y)) = (b - a, y / x)$. Then (TMSPS, IVLS, int) is a GIS. It is commutative. The interval $(b - a, y / x)$ measures our presumed sense that time-span (b, y) begins $b - a$ units after time-span (a, x) and lasts y / x times as long.

For example:



int $[a, b]$ = int [(1, 2), (3, 3)] = [3 – 1, 3 / 2] = [2, 1.5]
int $[b, c]$ = int [(3, 3), (5, 4.5)] = [5 – 3, 4.5 / 3] = [2, 1.5]

GIS replaces Lewin's time-span attributes with PAs, IVLS with PAIs, and employs the same definition for function *int*. TEMPS is a set of PAs. The PAI_1 between two successive PAs is then measured according to the function *int*, which encloses within an ordered pair the difference of the two PAI_1s' initial items and the ratio of their terminal items (as shown). The PAI_2 can then be derived in two ways: first, by applying the function *int* to the first and third PAs; or, second, by taking two successive PAI_1s, $[i, p]$ and $[j, q]$, and applying the group operation provided by Lewin: $[i, p][j, q] = [i + j, pq]$.[13] Since the ordering of PAI_1s can be

effectively. Since my analysis does not concern timespans, but rather with the grouping and speed of pulses, the commutative GIS is suitable.

[13]PAI is the value of the *int* function, and the commutativity of the GIS is determined by the commutativity of the binary composition in the group IVLS,

reversed without affecting the outcome of PAI_2, (TEMPS, PAIs, int) forms a GIS.[14] When the values for *a*, *b*, and *c*, are transferred to the tempo-span GIS, they produce effects that are inversely related to the timespan GIS: the values model a sequential *expansion* in the length of the timespans, while they model a rapid *contraction* (acceleration) in the speed of pulse in the tempo-span context.[15]

The analysis based on the tempo-span GIS is predicated on the notion that the metric modulations in Carter's works can be translated more effectively into a network of changing pulses and pulse streams. The chief advantage of this analytical method lies in its capacity to *quantify* the changes in the pulse organization in spite of the notated changes in meter and tempo. As the ensuing analysis of the timpani works shows, the pulse may be determined independently of the notated meter, based on contextual accents, *e.g.*, agogic, dynamic, register, contour, along with other contextual criteria invoked. The PAIs, in quantifying

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i.e., the addition for the first term of the PAI and multiplication for the second term of the PAI. PAI_1s and PAI_2s are both elements of PAIs; the only difference between them is their relation to the ordered succession of PAs.

[14]See Lewin, p. 26. He provides a definition of GIS as follows: a *Generalized Interval System* (GIS) is an ordered triple (S, IVLS, int), where S, the space of the GIS, is a family of elements, IVLS, the group of intervals for the GIS, is a mathematical group, and int is a function mapping S x S into IVLS, all subject to the two conditions (A) and (B) following.

> (A): For all r, s, and t in S, int (r, s) int (s, t) = int (r, t).
>
> (B): For every s in S and every I in IVLS, there is a unique t in S which lies the interval I from s, that is a unique t which satisfies the equation int $(s, t) = i$.

In my *tempo-span* GIS, PAIs form a mathematical group and the mapping of the function int of PAs into PAIs fulfills Lewin's conditions (A) and (B).

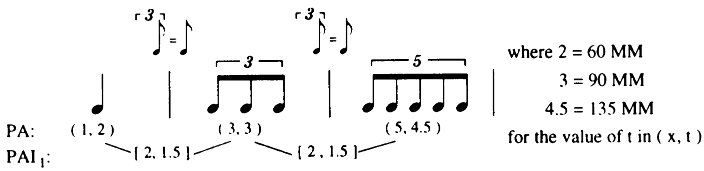
[15]The time-span GIS and the tempo-span GIS are inversely related in the general sense that the second digit of the ordered pair denotes the length of the time-span in number of time units in the former and the speed of the time unit in the latter: the larger the number, the time-span becomes longer, while the speed quickens in the tempo-span GIS, and vice versa. The first digit of the ordered pair, on the other hand, models perceptually different phenomena: in the time-span GIS, the first digit reflects the points of attack in a contextually defined time-point, while in the tempo-span GIS, the digit represents merely the number of subdivisions within a pulse.

the net changes in pulse grouping and speed, model the relative continuity and discontinuity in the temporal flow of Carter's music, as shall be discussed. In short, the tempo-span GIS provides a "middleground" reading in the sense that it focuses on the change in the *relationship* of grouping and speed of pulses rather than on the changes in the foreground rhythmic characteristics.

Figure 2b. Definition of a tempo-span GIS.

TEMPS is a set of pulse attributes (abbreviated PAs) of the form (x, t) where x is the number of subpulses contained by a pulse and t is the speed of a given pulse. The first-order pulse attribute interval (abbreviated PAI_1) between two successive PAs (x_1, t_1) and (x_2, t_2) is calculated following Lewin's function int, which maps S × S into IVLS: $int((x_1, t_1), (x_2, t_2)) = [x_2 - x_1, t_2 / t_1] = [i, p]$. The second-order PAI (abbreviated PAI_2) between PAs two apart, given PAI_1s $[i, p]$ and $[j, q]$, derives from the group operation $[i, p][j, q] = [i + j, pq]$. Because $[i, p]$ and $[j, q]$ can be interchanged, (TEMPS, PAI, int) forms a GIS of the commutative type.

For example:



Analysis of "Canaries"

"Canaries," the seventh etude from Carter's *Eight Pieces for Four Timpani*, refers not to "chirping" birds (though a pun may be intended), but to a Renaissance dance imported from the "wild men" of the Canary Island.[16] The timpani pitches are restricted to the tetrachord, E, B, C♯, F. It also features three

[16]Schiff, p. 150.

distinct timbral effects designated on the score as C, N, and R. C refers to "striking at the center of head," N to "playing in normal position," and R refers to "striking on the head very close to the rim." The dotted line above the score indicates a gradual transition from one playing position to another.

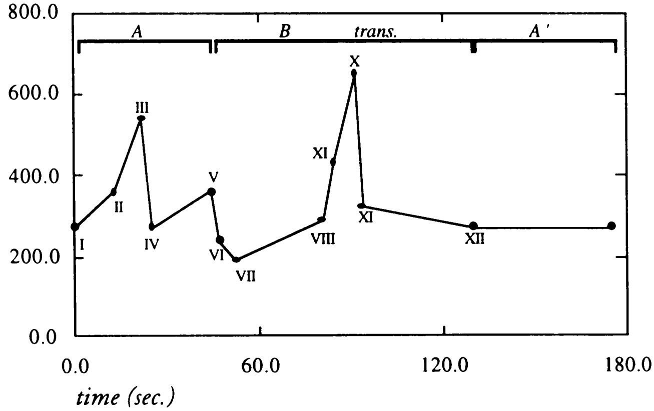
Figure 3a. Overview of changes in speed of pulse, ratio, and duration for "Canaries."

Section		*Speed of pulse*	*Comparison of speed*	*Ratio of change in tempo*	*Sectional duration (sec.)*	*Cumulative duration (sec.)*
A	I	♩. = 90	♪ = 270			12.67
	II	♩. = 120	♪ = 360	3:4 (25% faster)	8.50	
	III	♩. = 180	♪ = 540	2:3 (33.33% faster)	3.11	
	IV	♩. = 90	♪ = 270	2:1 (50% slower)	19.31	
	V	♩. = 120	♪ = 360	3:4 (25% faster)	3.00	46.59
B	VI	♩ = 120	♪ = 240	3:2 (33.33% slower)	5.00	
	VII	♩ = 96	♪ = 192	4:5 (20% faster)	30.00	
	VIII	♩ = 144	♪ = 288	2:3 (33.33% faster)	3.33	
	IX	𝅗𝅥. = 72	♪ = 432	2:3 (33.33% faster)	7.22	
	X	𝅗𝅥. = 108	♪ = 648	2:3 (33.33% faster)	2.22	48.17
trans.	XI	♩. = 108	♪ = 324	2:1 (50% slower)	36.38	
A'	XII	♩. = 90	♪ = 270	6:5 (16.67% slower)		45.33

The overall temporal structure of "Canaries" articulated by the metric modulations is shown under Figure 3a. Roman numerals denote successive changes in speed of the pulse that result from the metric modulations. Ratios of change in tempo are determined by dividing the speed of the subpulse of one section by the speed of the previous one. Figure 3b illustrates graphically the sequence of acceleration or deceleration in tempo that results from the metric modulations.[17] The horizontal axis depicts

[17] In the present context, the terms acceleration and deceleration do not indicate *continuous* changes, but rather *discrete* changes effected by metric modulations and other contextual means.

Figure 3b. Graphic illustration of change in eighth-note speed.

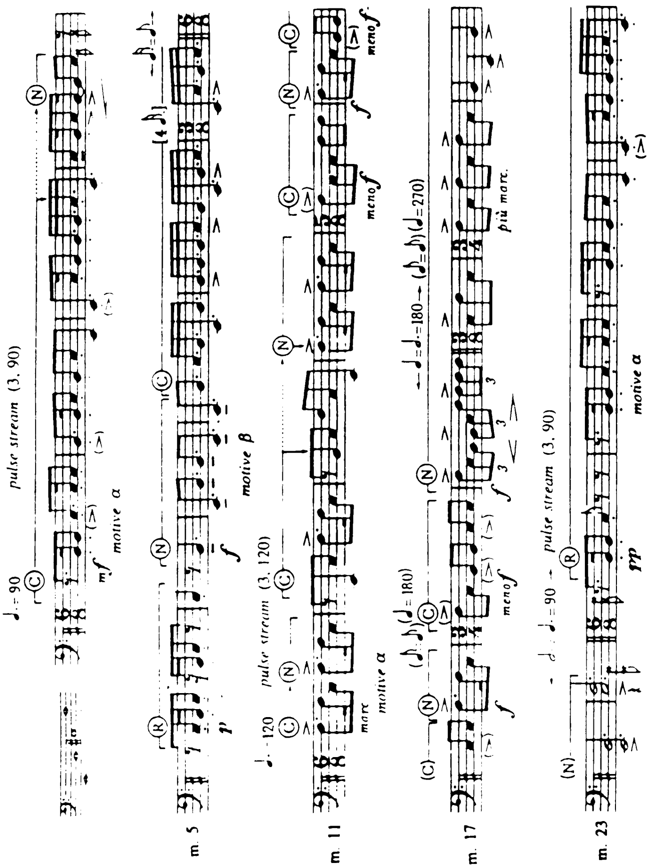


elapsed time in seconds, while the vertical axis depicts the change in the speed of an eighth note.

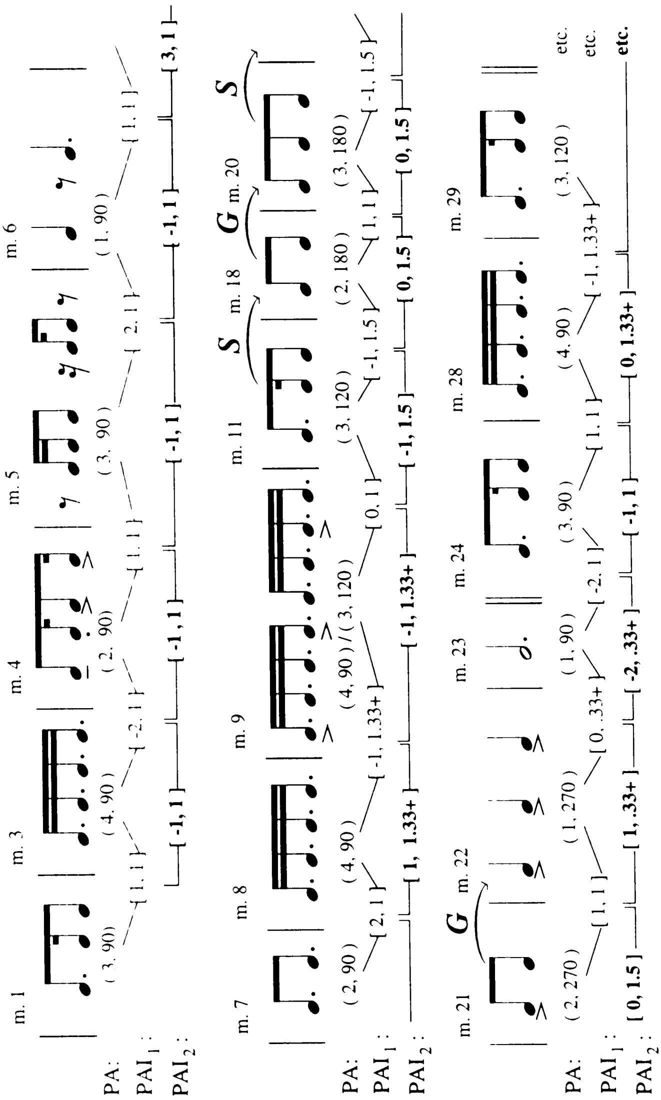
In the timpani works, Carter systematizes the metric modulations through prescribed ratios to regulate the overall continuity and formal balance. For instance, note how he uses the 2:3 and 2:1 ratios of tempo change in the A section, and then creates a sequence of acceleration based on the 2:3 ratio to reach the maximum speed of ♪ = 648MM at the end of the B section. The entry into each new section is always accompanied by a deceleration in tempo, and the ternary form is established through the reinstatement of the initial meter of 6/8 and speed of ♩. = 90MM.

Example 3a presents the score to the opening section of "Canaries." Example 3b presents my tempo-span GIS analysis of this passage. The opening section establishes a pulse stream through the reiteration of motive α, characterized by the B–C♯ motion with the triple grouping of the pulse. While the accents on the downbeats in mm. 2–3 project a strong 6/8 metric "feel" at the onset of the piece, this is offset by the countermotive β that initiates a duple grouping (featuring the fifth E–B) of the pulse at m. 7. This section traverses through several metric modulations before making a cyclical return to the initial pulse stream (3, 90) at m. 25.

Example 3a. "Canaries," section A (mm. 1–44).



Example 3b. Tempo-span GIS network for "Canaries," mm. 1–44.



The first metric modulation to PA (3, 120) at m. 11 is brought on by previous changes in the grouping of the pulse from triple, duple, to quadruple. Even by the end of the first system, the quadruple and the hemiola-like duple divisions create rhythmic friction against the PA (3, 90). At m. 9 Carter uses cross-accents to regroup the quadruple subdivision into a group of three dotted sixteenth notes. Example 3b shows this regrouping at m. 9, which creates an overlap of PAs (4, 90) and (3, 120) as indicated by the slash ("/"). The perception of pulse grouping is ambivalent at this point, analogous to the dual function of a pivotal chord in effecting a modulation in common-practice tonality. As indicated by the PAI_1, [–1, 1.33+], the speed increases by one-third, while the subdivision decreases from quadruple back to the triple subdivision. The sequence of PAI_2 shows a steady chain of [–1, 1] until m. 7, where the transition to the new tempo begins. Notice how the transition to the new pulse speed occurs ahead of the point at which the metric modulation is notated on the score (m. 11).

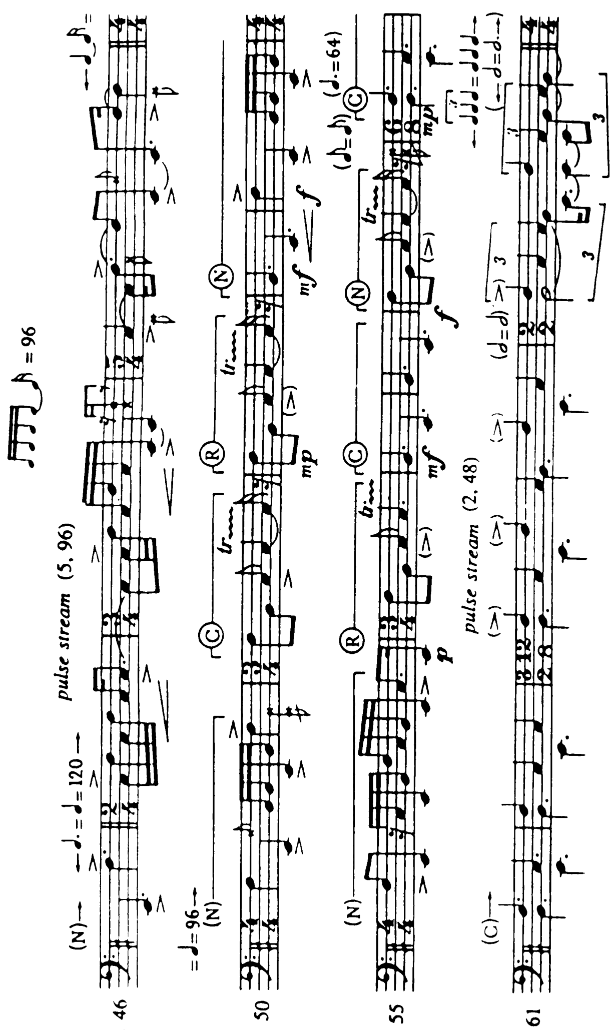
The next metric modulations occur sequentially between mm. 11 and 24. Here Carter alternates between two strategies to induce a sequential acceleration in tempo by a 2:3 ratio: (1) change the internal grouping of the pulse from two to three (no change in the speed of pulse); (2) truncate the subpulses from three to two (speed of pulse accelerates by 2:3). Referring to the first strategy as operation *G* and the second as operation *S* (as illustrated in Example 3b), these two operations are used alternately to induce a tempo acceleration from 120MM to 270MM in the course of five measures. We can see in Example 3a that this transition is mediated beginning at m. 15, where Carter first alters the pulse by truncating the 6/8 meter to 5/8, creating an alternating triple and duple pulse groupings within mm. 15–17. Then he introduces an exclusively duple grouping at m. 18 that induces the shift in PA from (3, 120) to (2, 180). PAI_1, [–1, 1.5], indicates that the speed of the pulse increases by one-third while the subdivision decreases by one. Next, he alters the grouping of the pulse from two to three at m. 19. PAI_1 [1, 1] shows how the number of subpulses has increased by one but the tempo of the pulse remains the same. At m. 21, the accents regroup the pulse structure from three to two to create another tempo acceleration of ♩ = 270MM. The sequential acceleration from 120MM to 270MM can therefore be modeled by the succession of PAIs [–1, 1.5] and [1, 1]. This results in a chain of identical PAI_2s, each **[0, 1.5]**.

At m. 23 the pulse slows down abruptly from PA (1, 270) to (1, 90). This juncture initiates the return of motive α at m. 25, and the abrupt tempo deceleration by 3:1 imparts a definitive "rhythmic" cadence. The timbral change to "playing near the rim" at m. 25 also produces a distant, echoing effect in alluding to the opening material. Carter then modulates to PA (3, 120) at m. 44 (just as he had in the beginning). The A section, therefore, presents a circular construction of tempo modulations.

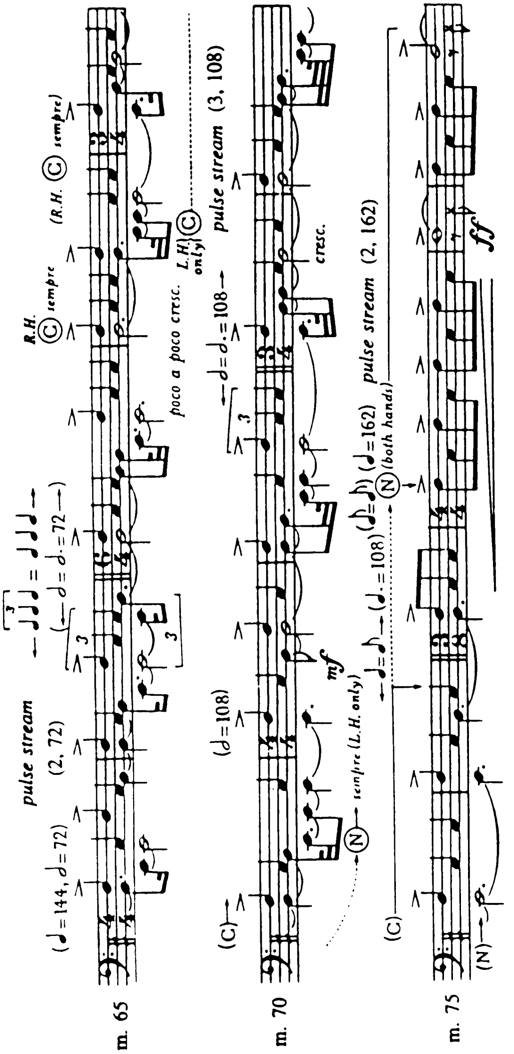
There are two significant features that characterize the middle section of "Canaries." The first has to do with the metric ambiguity that the listener experiences at the border of sections A and B. Despite the notational change in meter and rhythmic displacements that begins in m. 47, there is a pulse stream that becomes established contextually. As shown in Example 4a, the accents (>) regroup mm. 47–49 into five sixteenth-note pulse units so that a pulse stream (5, 96) is established. The metric modulation of ♩ = 120MM (at m. 5) and the changes in meter from 2/4 to 3/4 and 5/4 are, therefore, superfluous. Example 4b shows that the transition from the A section coincides with the shift from PA (1, 120) to (5, 96). The disjunction and the metrical ambivalence perceived at the onset of the B section is mirrored by the abrupt change that occurs in PAI_2 **[–3, 0.8]**. After the initial temporal "disjunction," Carter restores temporal continuity by alternating between two rhythmic patterns that lend symmetry in the PAI_1s (see the succession of [0, 1] in the GIS network).

The second significant feature of the middle section involves a sequence of tempo acceleration based on the 2:3 ratio. Beginning at m. 60, the timpani part divides into a counterpoint of two voices, where the lower voice provides an ostinato against the upper voice. While the notation suggests a complex rhythmic

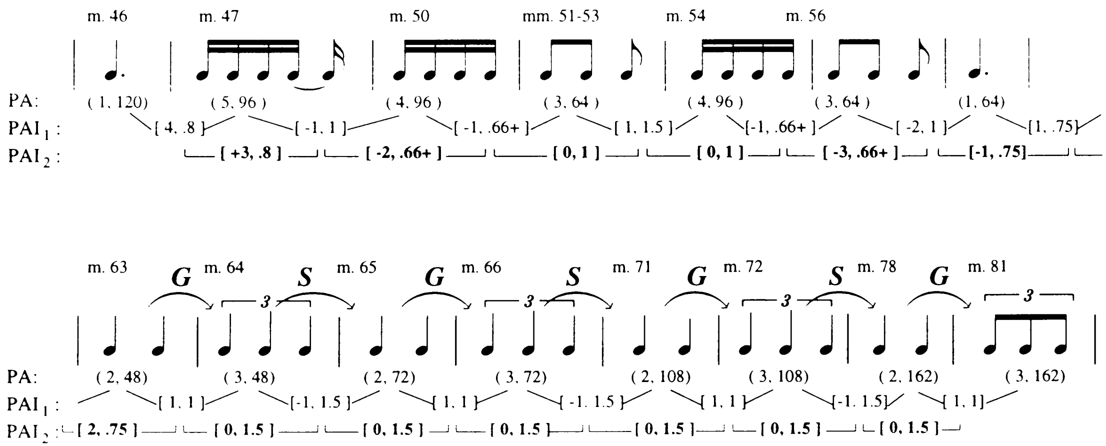
Example 4a. "Canaries," section B (mm. 46–84).



Example 4a (continued).



Example 4b. Tempo-span GIS network for "Canaries," mm. 46–88.



interaction between the two voices, this interaction can be described more simply as a counterpoint between a moving pulse (in the upper voice) that accelerates at the ratio of 2:3 against a steady pulse (where the lower voice remains at ♩ = 64MM). This trend culminates in the *ff* arrival on F at m. 79. The tempo-span GIS network in Example 4b shows how the sequential acceleration within mm. 63–81 is based on the same alternating sequence of grouping and speed (denoted by the letters *G* and *S*). These changes are identical to those found in mm. 11–21 of the A section: notice how the sequence of PAI2s, [0, 1.5], produces the sequential acceleration from PAs (2, 48) to (3, 162).

The entry on the whole-note F at m. 81 provides a palpable point of arrival. This passage begins a transition (leading back to section A' at m. 109) where the tempo stabilizes to 162MM and the pulse conforms to the notated 4/4 meter. The dynamic and accentual emphases given repeatedly to the note F help to sustain the climactic effect of this passage.

The significance of the A' section will be summarized briefly. Example 4c provides the tempo-span GIS analysis of the final section of "Canaries." In mm. 106–108, Carter induces a smooth transition back to the opening by altering the duple pulse grouping to a triple one, thus slowing down the tempo by the familiar ratio of 3:2 to bring the speed back to ♩. = 90MM and the pulse stream at PA (3, 90). While this concluding section contains not a single metric modulation, the network of the tempo-span GIS in Example 4c reveals a complex pattern of changes in the pulse organization. Carter avoids the establishment of a pulse stream and creates rhythmic tension and discontinuity through continuously changing meters. The second line of the GIS network illustrates how this passage intersperses PA (3, 180) between (3, 90) and (2, 135); this trend culminates in a cross-rhythm of 2:3 at mm. 129–131, where PA (3, 90) is superimposed over (2, 135). The resulting network of PAIs is highly discontinuous: the same intervals recur from time to time but without any regularity. While the 6/8 meter is restored at m. 132, Carter continues to generate rhythmic tension to the end

Example 4c. Tempo-span GIS network for "Canaries," mm. 106–end.

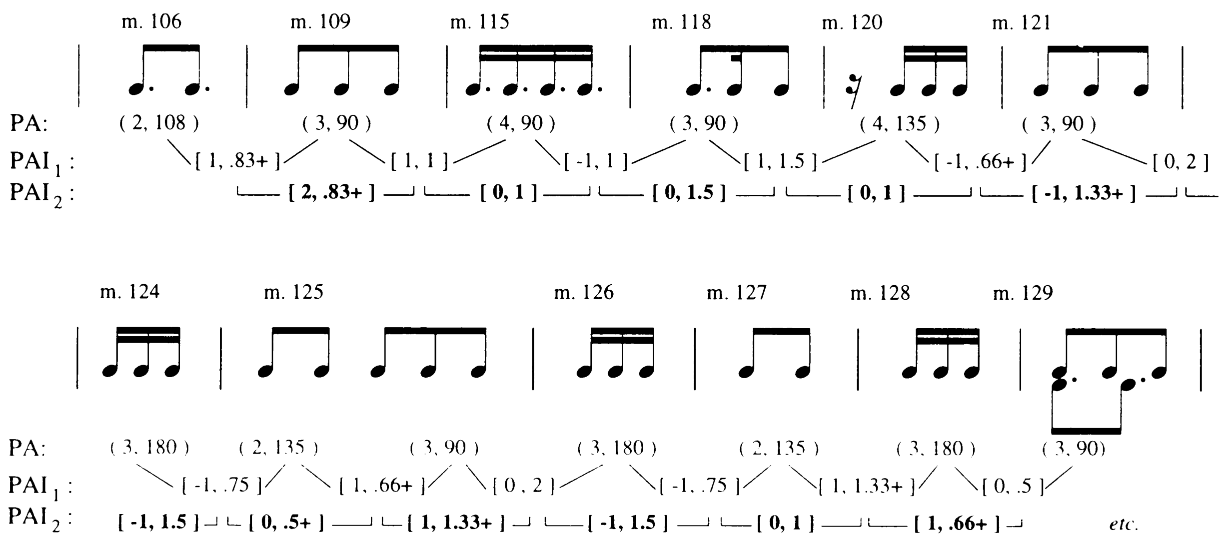
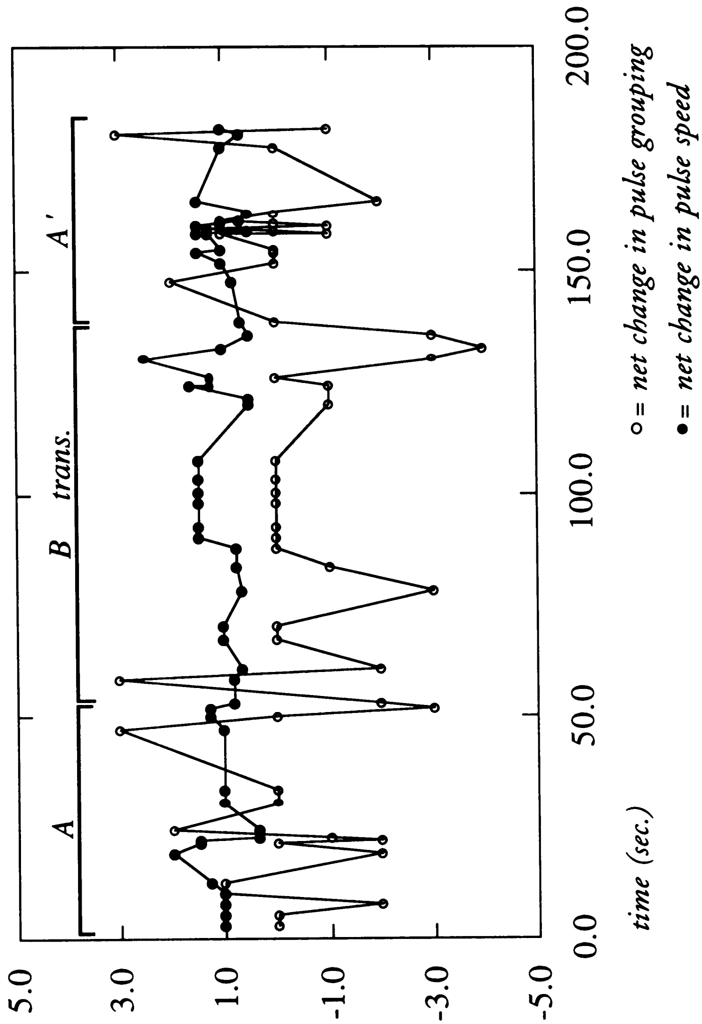


Figure 3c. Net intervallic changes in the pulse organization of "Canaries."



through hemiola-like alternation between the duple and triple pulse grouping and through the accentual displacement of beats.

In this way, the tempo-span GIS network reveals how the rhythmic fluidity in this work derives from the systematic changes in the grouping and speed of pulses: metric modulations serve merely as landmarks in the continuous stream of changing pulses. To model the overall temporal continuity, Figure 3c graphs the successive PAI2s formed in the course of "Canaries." The horizontal axis depicts elapsed time in seconds, while the vertical axis depicts the net changes in the grouping and speed of pulse. The sectional boundaries are indicated in brackets above the plotted lines.

Notice how the temporal discontinuity at formal boundaries is modeled by the abrupt fluctuations in the intervallic changes in the grouping of pulses; the decrease in rhythmic momentum at the onset of B section corresponds to the negative intervallic trend while the increase in rhythmic momentum in approach to the A' section is accompanied by a positive trend. The condensed web of plotted lines in the A' section depicts the highly discontinuous changes in the pulse organization of the final section that could not be ascertained by examining the metric modulations alone. On the contrary, the steady rate of acceleration that generates continuity in the B section is modeled by the plateau that is formed in the middle of the graph. The form is thus balanced by the symmetry in the PAIs that lends relative continuity and stability *within* sections, and asymmetry or abrupt changes in the PAIs that lends discontinuity notably at formal junctures.

Analysis of "March"

"March," the last etude of this series, features a polyphonic interaction of two "drummers" played by a solo timpanist. Schiff comments as follows:

The last piece is a very Ivesian jest, which some drummers also hear as a portrait of the New York Philharmonic's timpanist for fifty years—his conservative manner may be reflected in the one-fifth tuning of the piece. There are two marches, each at its own speed, one played with the heads of the sticks, the other with the butts. The shape of the piece suggest a hypothetical scenario which only those who played in an American marching band could understand. Two drummers approach each other playing at different speeds. They meet and 'challenge' each other, imitating each others' figures and outdoing one another in virtuosity. Having established their equal credentials, they then march away at different speed.[18]

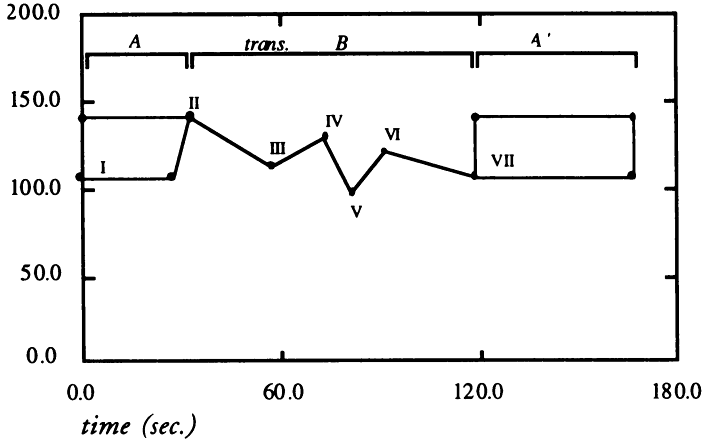
The pitches for this piece are restricted to the tetrachord G–B–C–E, and the competition between the two "drummers" is portrayed by the timpanist playing one part on the head and the other part on the butt of the instrument. Figures 4a and 4b provide a list of metric modulations in "March" and the corresponding graphic illustration. As in "Canaries," the ternary form is articulated by the systematic return of the opening tempi and pulse structure. The metric modulations involve three ratios and their inverses: the pattern of acceleration by 3:4, deceleration by 5:4, acceleration by 7:8, and from section V, the ratios appear in inverse to arrive at the original tempo in the final A' section. While the scheme of metric modulation is simpler than that of "Canaries," the texture is complicated by the polyphonic interaction of two pulses (♩ = 105MM vs. 140MM) in the outer sections. The middle section, which depicts the intense rivalry

Figure 4a. Overview of changes in metric modulation, ratio, and duration for "March."

Section		*Metric modulation*	*Speed*	*Ratio of change in tempo*	*Sectional duration (sec.)*	*Cumulative duration (sec.)*
A	I (m. 1)	♩ = 105	♩ = 105			33.26
trans.	II (m. 15)	♩ = 140	♩ = 140	3:4 (25% faster)		24.43
B	III (m. 29)	𝅗𝅥 = 56	♩ = 112	5:4 (20% slower)	16.07	
	IV (m. 37)	𝅗𝅥 = 64	♩ = 128	7:8 (12.5% faster)	7.05	
	V (m. 46)	𝅗𝅥 = 48	♩ = 96	4:3 (25% slower)	9.69	
	VI (m. 47)	♩ = 120	♩ = 120	4:5 (20% faster)	27.50	84.74
A'	VII (m. 59)	♩ = 105	♩ = 105	8:7 (12.5% slower)		47.43

[18] Schiff, p. 151.

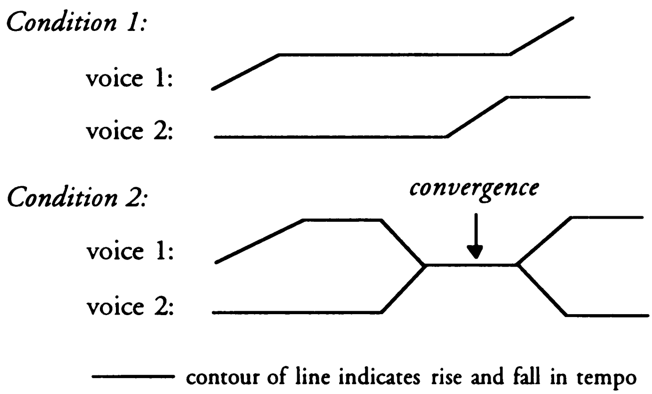
Figure 4b. Graphic illustration of change in quarter-note speed in "March."



between the two drummers, presents continual fluctuations in the speed and grouping of the pulse.

The polyphonic texture in the outer sections calls for modification in the criteria for constructing the tempo-span GIS network. Based on textural changes that occur in "March," two conditions are stipulated in Figure 4c. If two voices unfold

Figure 4c. Tempo-span GIS network for polyphonic textures.



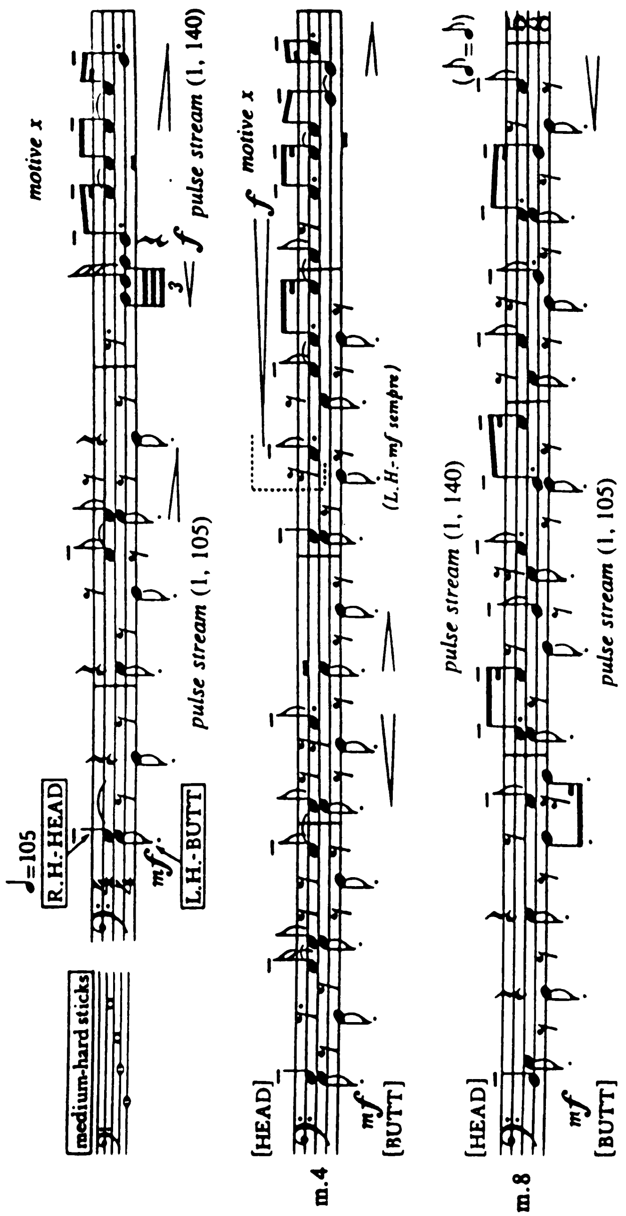
polyphonically without convergence (condition 1), then the GIS networks are constructed separately for each voice to account for the temporal fluctuations that occur in the successive dimension. The composite differences in the speed between the two voices may also be calculated as ratios in the simultaneous dimension. If the voices merge (condition 2) and the pulse organization changes at the textural boundary, these changes are considered to affect both voices and the PAIs are computed for both parts proceeding into and out of the composite segment.

Example 5a shows the score and Example 5b shows the rhythmic reduction and the tempo-span GIS network for the A section of "March." The opening three measures are marked by the steady quarter-note pulses in the lower voice that establish a pulse stream at PA (1, 105). When the texture collapses to a monophonic segment in the upper voice at m. 3, there is a shift from PA (1, 105) to PA (1, 140) in the successive dimension. Since the lower voice resumes (1, 105) by m. 4, the alternation between (1, 105) and (1, 140) creates a sequence of PAI_1 that yields the PAI_2 [0, 1]. The rhythmic figure in m. 3 is designated as motive *x*, since it provides repeated closure at phrase endings in the opening section of "March." The upper voice, in comparison, reveals a more frequent and erratic change in the speed of pulse. It begins with the PA (1, 30) that is weakly established through one repetition of this timespan, then changes abruptly to PA (1, 140) at m. 3. In mm. 4–8, the tempo modulates sequentially from 60MM to 140MM, modeled by an accelerating trend in PAI_1 from [0, 0.42+], [0, 1.4], to [0, 1.66+].

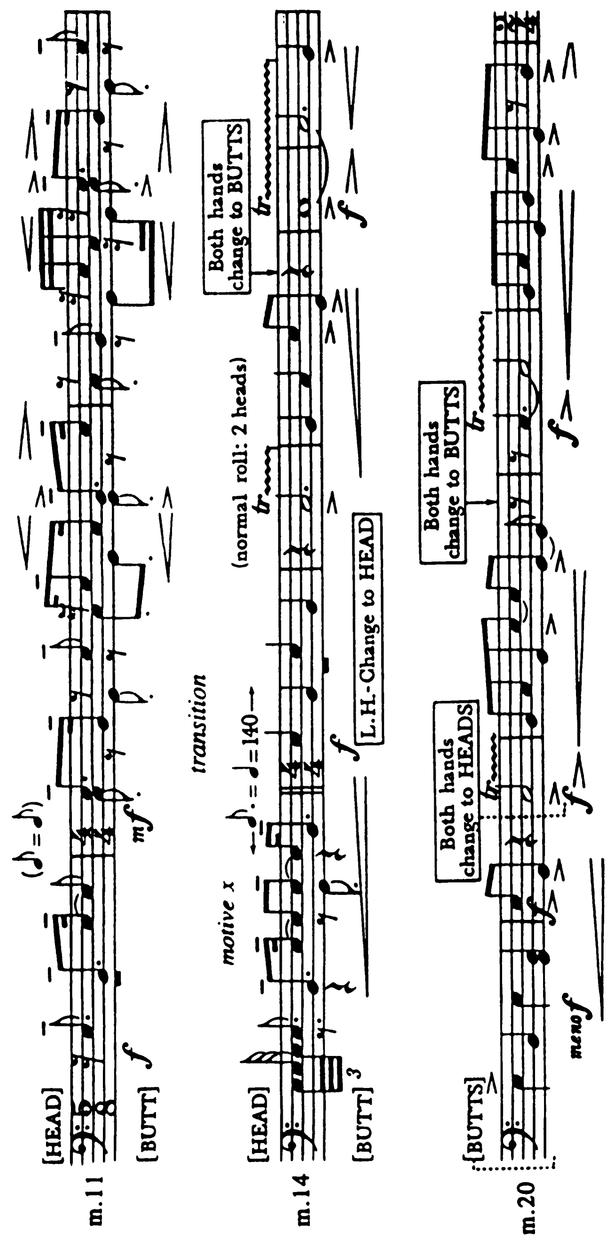
Beginning in m. 4, the lower voice maintains PA (1, 105) but the polyphonic interaction projects another pulse organization of PA (2, 52.5). This ambivalence stems from the rhythmically displaced entry of the upper voice against the recurring C–G in the lower voice (see Example 5a, m. 5 on): the pitch C becomes accentuated as a contextual "downbeat" so that the half-note timespan tends to be heard as a *pulse* instead of the quarter note.

Beginning in m. 8 (the third system), the two voices "lock in" so that the PA (1, 140) is superimposed over (1, 105) in the lower voice. The melodic and rhythmic contours organize the pulses

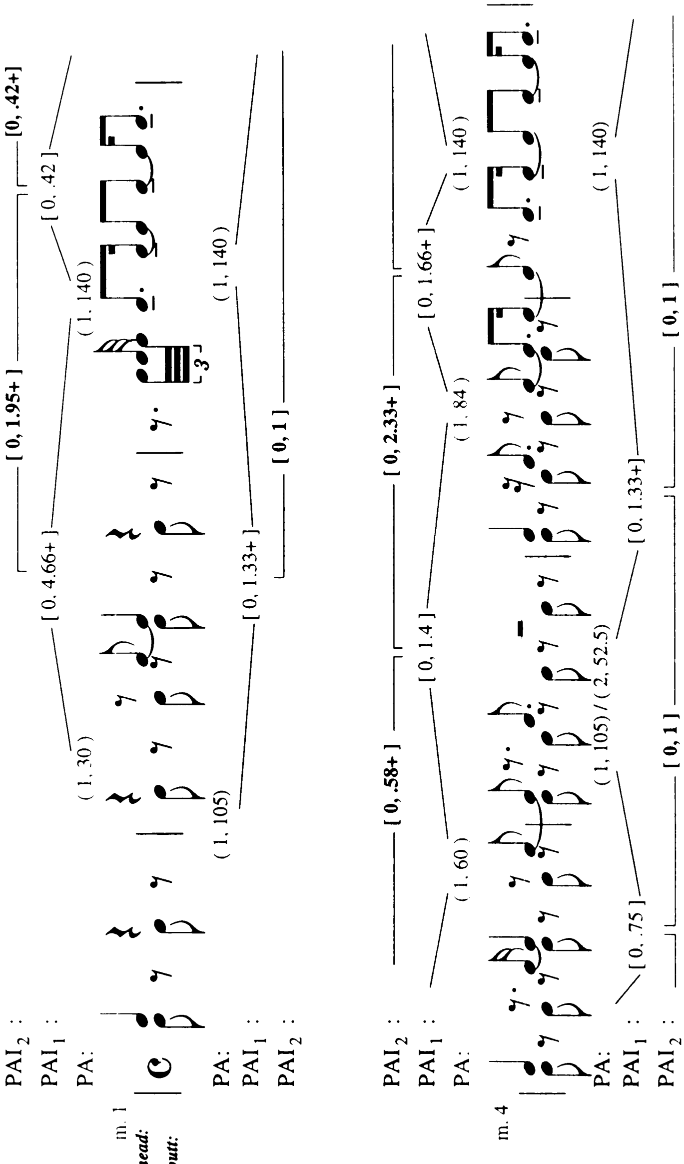
Example 5a. "March," section A and transition (mm. 1–24).



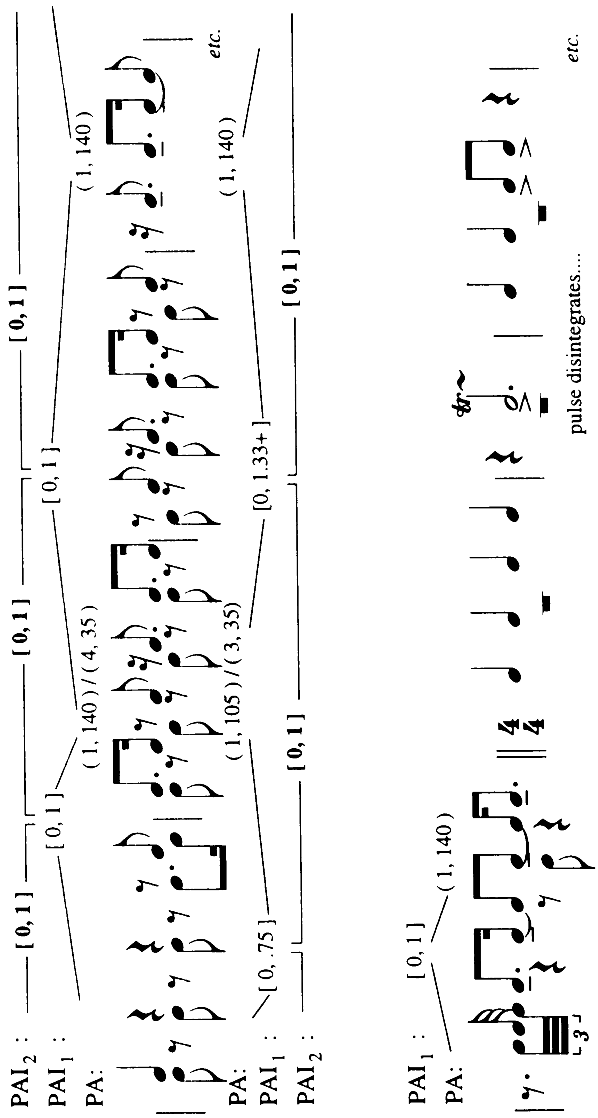
Example 5a (continued).



Example 5b. Tempo-span GIS network for "March," mm. 1–14.



Example 5b (continued).



into PA (4, 35) against (3, 35). This establishes cross-pulses related by the 3:4 ratio in the simultaneous dimension.[19] Carter thus unifies the A section by exploiting the 3:4 ratios in the successive (105MM/140MM) and in the simultaneous (140MM/105MM) dimensions. This texture continues until the first metric modulation occurs at m. 14, using motive *x* simultaneously as a phrase ending and as a link to the transitional passage that ensues.

As in "Canaries," the transition to the B section is marked by a liquidation of the prevailing pulse stream. The texture simplifies as the passage features an alternation in the soloistic display of one drummer to another. While the notated meter denotes 4/4, the foreground rhythm fails to reinforce a quadruple grouping. For instance, the trill figure on C, creates a strong accent and a contextual "downbeat" and enters at irregular time-span intervals in ways that fail to define any type of pulse.

Example 6 provides the rhythmic reduction and the tempo-span GIS analysis for the middle section of "March." The frequent modulations found in course of the B section can be categorized into three distinct operations: (1) increasing the number of subpulses to slow down the speed of the pulse (operation $S\Downarrow$ with arrow down); (2) decreasing the number of subpulses to quicken the speed of the pulse (operation $S\Uparrow$ with arrow up); (3) changing the grouping of *pulse* without changing speed (operation *G*).

With respect to overall tempo changes, notice that there is a pattern of deceleration (featuring operations $S\Downarrow$ and *G*) between mm. 25–48, followed by a pattern of acceleration (featuring operation $S\Uparrow$) toward the return of the A section. More specifically, at mm. 25–26, the PA shifts from (4, 70) to (5, 56), causing a deceleration in tempo by a 5:4 ratio, or 0.8. At m. 35, the next notable transition begins, where the tempo abruptly dips from 112MM to 64MM through an elongation of the pulse subdivision from four to seven sixteenth notes; this creates the shift in PAs from (4, 112) to (7, 64) and the disjunct shift from PAI_2 of

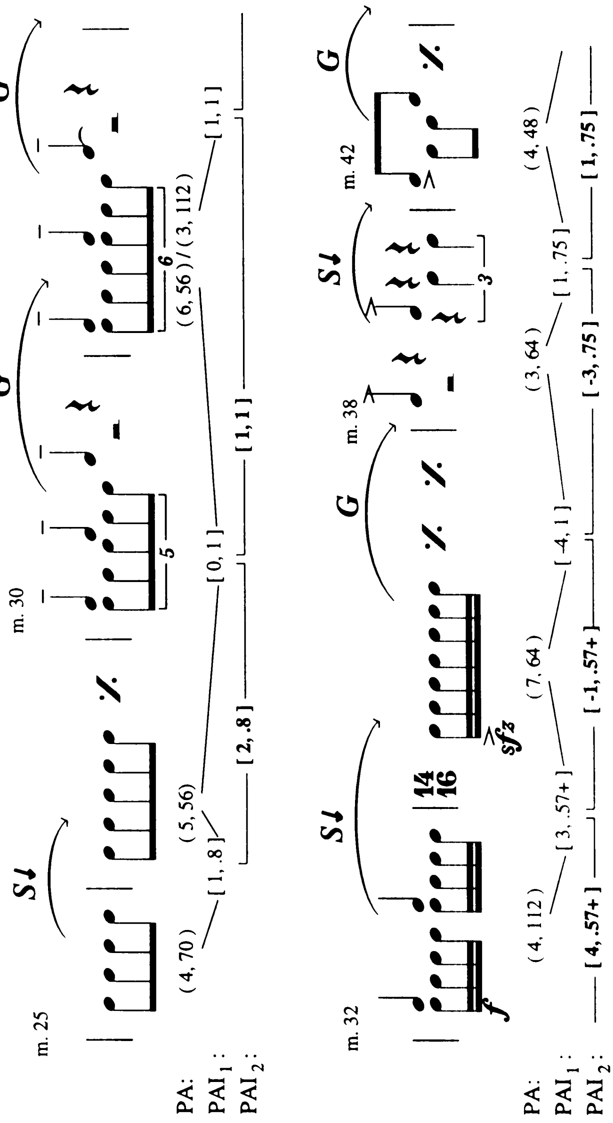
[19]An alternate interpretation may view the two parts as sharing the same duple *pulse* but proceeding at different tempi: upper voice in (2, 70) and the lower voice in (2, 52.5).

[1, 1] to [4, 0.57]. A further deceleration in tempo occurs at mm. 42–44, where the elongation of a triplet to a quadruple subdivision generates the shift from (3, 64) to (4, 48). The slowest point is reached at mm. 46–48 where the PA drops from (5, 48) in mm. 44–45 to (4, 30) in the preceding measures.

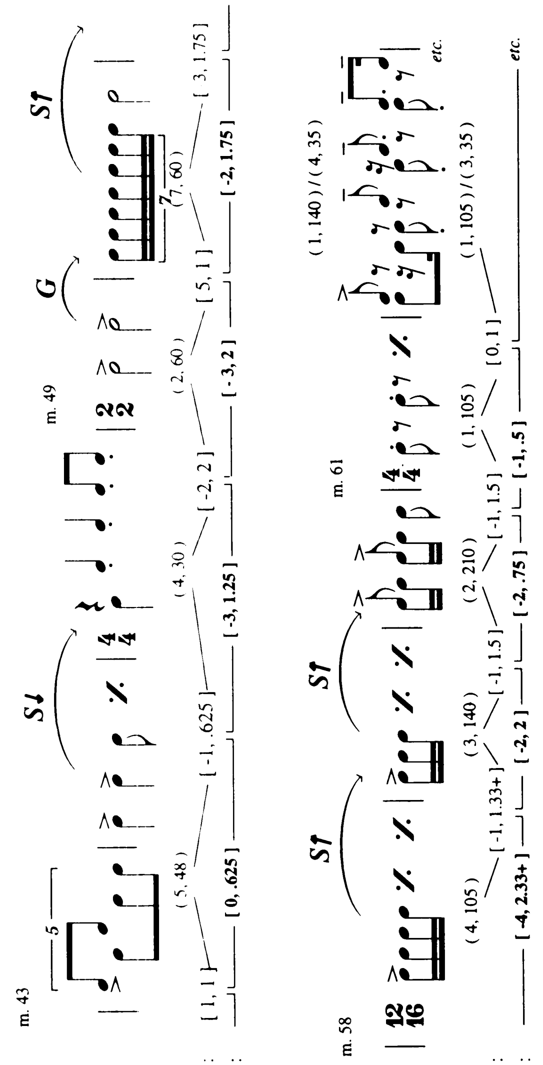
From this point on, there is a gradual acceleration in tempo created through rhythmic truncation. Following m. 48, the 2/2 meter sets the half note as a pulse at (2, 60), resulting in a tempo acceleration by 2. This is followed by a truncation of a septuplet subdivision of the half-note beat, (7, 60) to quadruple subdivision at m. 58. To build further rhythmic momentum toward the return of A, Carter creates an acceleration scheme of PAs (4, 105), (3, 140), and (2, 210), through a successive truncation of a sixteenth note. The septuplet at m. 50 is truncated by m. 58 to a quadruple grouping (with the speed of the sixteenth note held in common), then successively to a triple and a duple grouping, generating PAIs that increase in tempo sequentially up until m. 60. Notice how Carter skillfully incorporates the two tempi integral to the A section, 105MM (m. 58) and 140MM (in the following measure), within the chain of acceleration in preparation for the return to the opening section.

In "March," therefore, Carter employs two distinct strategies in balancing the form: on the one hand, stability and continuity in the outer sections are generated by maintaining the pulse stream that involves the polyphonic interaction of two pulses at 105MM and 140MM; instability and discontinuity in the middle section, on the other hand, are generated through truncating and elongating the number of subpulses to change the grouping and the speed of the ongoing pulse. Figure 4d illustrates the graphic changes in PAI_2 for "March." The outer sections are characterized by tempo changes (primarily in the upper part, as illustrated by the zigzagging pattern) that involve little or no change in the pulse grouping. The middle section, in comparison, is marked by notable fluctuations in the grouping of pulses. Unlike "Canaries," the final section of "March" reestablishes the polyphonic interaction between PAs (1, 140) and (1, 105) and

Example 6. Tempo-span GIS network for "March," mm. 14–64.



Example 6 (continued).



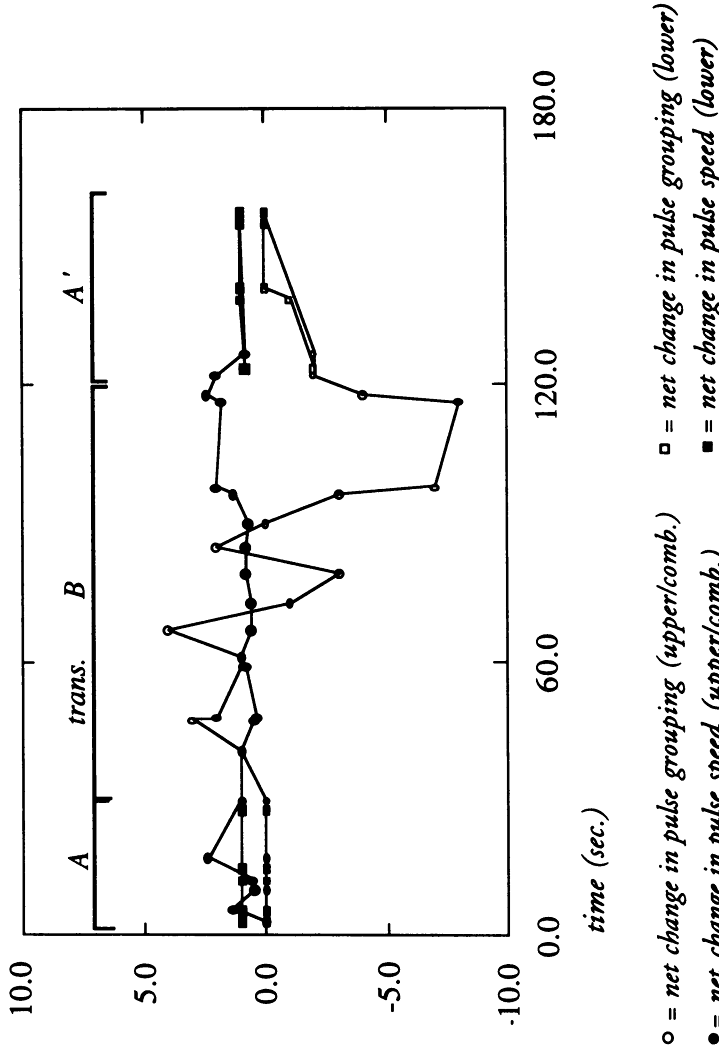


Figure 4d. Net intervallic changes in the pulse organization of "March."

maintains this texture to the end of the piece without further disruptions in speed or grouping of the pulse stream.

With respect to the overall formal balance, the main source of tension and discontinuity in "March" is found in the process of truncation and elongation of pulses that occurs in the *successive* dimension. This process is particularly prominent in the middle section of the piece. The cross-pulses in the outer sections, though metrically ambivalent, provide stability and continuity in the outer sections by unifying the temporal relationships in both dimensions through the 3:4 ratio.

Evaluation and further considerations

The tempo-span GIS measures the temporal continuity and discontinuity based on the additive change in the grouping and the multiplicative change in the speed of the pulse. It focuses on the change in the *relationship* of grouping and speed of pulse rather than on the qualitative changes in notated rhythms, motivic and gestural associations, *etc*. It also assumes that the perception of the temporal continuity in Carter's music is *not* metrically based, but based rather on transformations in the network of pulses and pulse streams. Even if there are passages where the meter can be inferred (where the tempo remains steady and contextual accents are synchronous with metric accents), metric hierarchy cannot be established in the context of constantly shifting tempi. These assumptions reflect how we perceive tempo changes which are primarily relational (slower or faster with respect to the preceding tempo) as opposed to absolute.[20]

In these early works from the 1950s, Carter employs metric modulations to induce sequence of acceleration or deceleration in tempo within the ternary formal archetype: acceleration generates tension and momentum toward a point of arrival, while deceleration imparts a sense of resolution at formal junctures.

[20]In other words, we do not generally perceive changes in tempo according to change in metronomic markings (*e.g.*, from 90MM to 120MM); rather, we perceive the change proportionally (*e.g.*, two-thirds faster).

One may even stipulate that the formal layout of metric modulations presents a temporal analogue to a modulation in common-practice tonality, marked by a systematic deviation to and from a *normative* pulse stream. Notated changes in meter are often rendered superfluous by contextual accents, *e.g.*, rhythmic grouping, melodic contour, that articulate the pulse or pulse stream.[21] In short, pulse, not meter, serves as the active agent in instigating the temporal modulations.

The present discussion of the tempo-span GIS, however, provides only a preliminary method for codifying the function of pulse in post-tonal music. It is designed for application to such works as Carter's timpani pieces, in which the non-rhythmic musical parameters are subsidiary to the function of pulse in regulating the temporal continuity. In considering extensions and modifications to other musical repertory, one must consider (1) factors that define the salient pulse attributes (PAs), which may not be limited to the speed and grouping of pulse as in the present analysis, and (2) a system for weighing pertinent musical parameters, *i.e.*, pitch, rhythm, timbre, dynamics, *etc.*, in determining context-specific criteria for PA segmentation.[22] In metrically ambivalent or ametric musical contexts, multiple interpretations of PA segmentation may be generated on the basis of differing perceptions of pulse.[23]

[21]For instance, key signature changes in the *Adagietto* from Mahler's *Symphony No. 5* do not necessarily reflect shifts in tonality commensurate with the key signature. The effect of metric modulation is analogous to enharmonic modulation in common-practice tonality in that the notation appears to be more complex than the aural effect of the transition.

[22]In the context of Carter's timpani works, rhythmic grouping, melodic contour, and modes of accentuation were considered primary factors in determining the PA segmentation.

[23]In the oft-discussed second movement of Webern's *Variations for Piano*, Op. 27, for example, conflicting PA segmentations may arise from weighting the structural importance of an eighth-note rest as a point of attack or an anacrusis. Based on rhythmic partitions alone, one PA segmentation (for mm. 1–8) involves a succession of PAs, (3, 106.66+) interspersed by PA (4, 80) based on hearing each eighth-note entry as the beginning of a PA and every eighth-note rest as an anacrusis. An alternate PA segmentation generates an alternating duple (2, 160) and quadruple (4, 80) succession of PAs based on hearing each

Further consideration should also be given to contexts that involve continuous changes (*i.e.*, gradual *accelerando* or *decelerando*) and polyphonic application of the tempo-span GIS analysis beyond two parts. Other temporal criteria, such as timespans of pulses and proportions of local and global sections, may be incorporated into the GIS analysis to present a more comprehensive analysis of temporal continuity and formal balance.

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eighth-note rest projecting a putative "downbeat." It is likely, in my opinion, that the performer may learn to play the piece according to the second PA interpretation (against the putative 2/4 meter), but the listener may potentially hear the piece according to the first PA interpretation or according to some other segmentation. The two interpretations reflect the dichotomy between the notated vs. "perceived" metric interpretations addressed by David Lewin in "A Metrical Problem in Webern's Opus 27," *Journal of Music Theory* 6/1 (1962): 124–132.