Comparing Collections Of Set Classes: Indexes In Forte's Genera Theory

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While pcset theory is well suited for measuring relationships between set classes or collections of set classes, the literature reveals that theorists question the numbers: what do the numbers mean and what assumptions do they serve? A case in point is the literature devoted to similarity relations, which spans almost four decades.[1] Among recent contributions to those ideas is that of Eric Isaacson, who provides a detailed critique of various approaches to the problem of similarity relations and offers a solution of his own.[2] The present essay proceeds in a similar spirit, but is concerned with relationships between *collections* of set classes rather than set classes themselves. The collections are those described by Allen Forte's theory of pcset genera.[3] I examine the

[1]See, for example, David Lewin, "Re: Intervallic Relations Between Two Collections of Notes," *Journal of Music Theory* 3 (1959): 298-301; Howard Hanson, *Harmonic Materials of Modern Music* (New York: Appleton-Century-Crofts, Inc, 1960); Donald Martino, "The Source Set and Its Aggregate Formations," *Journal of Music Theory* 5 (1961): 224-73; Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973); David Lewin, "Some New Constructs Involving Abstract Pcsets, and Probabilistic Applications," *Perspectives of New Music* 18 (1979/80): 433-44; John Rahn, "Relating Sets," *Perspectives of New Music* 18 (1979/80): 483-502; Robert Morris, "A Similarity Index for Pitch-Class Sets," *Perspectives of New Music* 18 (1979/80): 445-60; Michael Buchler, "Relative Saturation of Subsets and Interval Cycles as a Means for Determining Set-Class Similarity" (Ph.D. diss., University of Rochester, 1998). Of the various similarity relations, Allen Forte's are perhaps the most widely known.

[2]Eric Isaacson, "Similarity of Interval-Class Content Between Pitch-Class Sets: the IcVSIM Relation," *Journal of Music Theory* 34/1 (1990): 1-28.

[3]Allen Forte, "Pitch-Class Set Genera and the Origin of Modern Harmonic Species," *Journal of Music Theory* 32 (1988): 187-270. The first to develop a theory of genera was Richard Parks, in his *The Music of Claude Debussy* (New Haven: Yale University Press, 1989), which Forte acknowledges in "Round Table: Response and Discussion" (*Music Analysis* 17/2 (1998): 227-36), 230. Parks explores the commonalities and differences between his and Forte's

indexes Forte uses to measure relationships between genera and musical works as well as those between the genera themselves, and suggest a means of modeling more precisely the generic affiliation of set classes. Analyses of works by Webern and Schoenberg demonstrate the implications of my suggestion.

The twelve genera in Forte's theory range in size from twenty-one to sixty-five set classes, and they derive from trichords—or trichordal pairs—which are established on the basis of their uniqueness with respect to interval-class structure.[4] While the genera are formed principally by inclusion relations, which tend to proliferate, Forte limits genus memberships by imposing two further restrictions:

> Each member of the genus as well as its complement must be a superset of (must contain) the [trichordal] progenitor(s)....[Rule] 2. In addition to satisfying Rule 1, each pentachord must contain at least one of the tetrachords in the genus and each hexachord must contain at least one of the pentachords and at least one of the tetrachords in the genus.[5]

The second rule ensures a traceable "path" between any genus member and its progenitor. Even so, while some set classes hold membership in only one genus, most are members of more than one genus and a few belong to as many as eleven of the twelve genera. Set classes that cross generic boundaries, Forte notes, require special attention in analysis, particularly those that are members of most of the genera. He writes as follows of six hexachords, each of which belongs to eleven genera:

> Again, we can foresee an analytical problem here: if one of these gregarious hexachords turns up in a composition it is very apt to belong to every genus

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theories in his article, "Pitch-Class Set Genera: My Theory, Forte's Theory," *Music Analysis* 17/2 (1998): 206-226.

[4]Craig Ayrey, in his article, "Berg's 'Warm die Lüfte' and Pc Set Genera: A Preliminary Reading" (*Music Analysis* 17/2 (1998): 163-76), notes on pg. 176 the following corrections to Forte's genera: added to Genus 2 is set class 5-15; to Genus 3, set classes 5-32 and 6-33; to Genus 4, set class 6-16. In this article I use the corrected data.

[5]Forte, "Pitch-Class Set Genera," 192.

represented, which, once more, implies that strategies of interpretation will have to be developed to ensure a meaningful reading of generic organization.[6]

It is the set class that crosses generic boundaries that brings a measure of indeterminacy to generic analysis. While strategies of interpretation are crucial to the instantiation of the theory, one might approach the problem from another angle, and that is to consider how set classes are represented by the *indexes* that define generic relationships.

Of the two indexes Forte establishes, the *difference quotient* (or *Difquo*) and the *status quotient* (or *Squo*), the latter plays the more crucial role in analysis. The Squo determines the relatedness of a particular genus to any given collection of set classes (such as those that serve as resources for an actual composition) and is thus central to the instantiation of the genera theory. (For the sake of clarity, I will call the given collection of set classes the *set-class inventory*.) Forte's expression is as follows:

$$\text{Squo(GA)} = ((X / Y) / Z) \times 10$$

where GA denotes *Genus* A, X the number of set classes that intersect *Genus A* and the set-class inventory, *Y* the size of the set-class inventory, and *Z* the size of *Genus A*.[7] Insofar as it concerns generic interpretation, the higher the Squo of a given genus, the more closely that genus represents the set-class inventory.

The results of a generic analysis are displayed in a table called the *genera matrix* (see, for example, Table 1). The x-axis of the matrix lists the twelve genera, the y-axis the set-class inventory. An "o" marks each intersection of a set-class inventory member and a genus. Five "Rules of Interpretation" eliminate from the complete matrix those genera that do not contribute substantively to the profile of the set-class inventory,[8] and at this interpretative stage the Squo plays the key role. The possibility of

[6]Ibid., 209.

[7]Ibid., 232.

[8]With few exceptions, the interpretive rules invoke genera in descending order of their Squo values until all set classes of a composition are accounted for in the reduced genera matrix. See Forte, "Pitch-Class Set Genera," 234.

genus with a lower Squo overtaking one with a higher Squo appears only under one condition: genus intersection. That is, if all the matrix representatives of the genus with the higher Squo also intersect a genus with a lower Squo, and the latter genus has more matrix representatives than the former, then the genus with lower Squo prevails. Since one rarely encounters this condition, in most contexts the Squo alone serves as the agent of interpretation.

As the sole interface between compositions and the twelve genera, the Squo invites careful examination. Consider how it deals with an inventory in which most of the set classes cross many generic boundaries. Take, for example, set classes 4-1, 6-Z11, 6-15, 6-21, 6-22, 6-31, and 6-34. The hexachords in this inventory are the ones Forte so aptly calls "gregarious"; each holds membership in eleven genera. Set class 4-1, on the other hand, belongs to only one genus; Forte calls set classes that are attached to only one genus "singletons." The distribution of these set classes among the genera and the respective Squos of the genera are shown in Table 1. Each genus intersects at least four hexachords from the set-class inventory, while only "chromatic" Genus 5 intersects set class 4-1. That Genus 5 is the most representative of the set-class inventory seems self-evident: not only does it contain four of the six hexachords, but more importantly, it also contains the one set class of singular generic affiliation, 4-1. Given that the gregarious hexachords are essentially as indicative of one genus as of any other, their ability to engage a particular genus to the exclusion of all the others is negligible. In this generically ambiguous context of the hexachords, the sole genus engaged by set class 4-1 stands out in contrast to the others, and one might well expect the Squo to represent this fact. The Squo, however, interprets Genus 4 (the "augmented" genus) to be the most representative of the set-class inventory, even though all five hexachords it intersects appear together as members of eight other genera (along with the sixth hexachord, 6-Z11).[9]

[9]I set these six "gregarious" hexachords against the "singleton," 4-1, to highlight the imbalance of generic identity between set classes. Later we shall see the analytic ramifications of this imbalance.

Table 1. Complete Matrix and Squos of a Sample Case

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
4-1					o							
6-Z11	o	o	o		o	o	o	o	o	o	o	o
6-15	o	o	o	o	o	o	o	o	o	o		o
6-21	o	o	o	o	o	o	o	o	o	o		o
6-22	o	o		o	o	o	o	o	o	o	o	o
6-31	o	o	o	o		o	o	o	o	o	o	o
6-34	o	o	o	o		o	o	o	o	o	o	o
Counts:	6	6	5	5	5	6	6	6	6	6	4	6

Squos in Descending Order:

G4:	.340
G5:	.246
G8, G9, G10:	.209
G11:	.197
G6, G7, G12:	.190
G3:	.159
G1:	.136
G2:	.132

The Squos listed in Table 1 show how the various sizes of the genera influence the expression: for the most part the rankings accorded the twelve genera correlate in inverse proportion to size (only the rankings of Genus 11 and Genus 3 deviate from this pattern). The Squo formula prefers small genera, which is why Genus 4—with only twenty-one members—heads the list, and why Genus 2, with its sixty-five members, falls at the end.[10] Table 2 reduces the complete matrix of Table 1 following the five

[10]The smaller the divisor *Z*, which represents genus size in the Squo formula, the greater the quotient. Christian Kennett, in his article "Take Me Out to the Analysis Conference: Sets, Stats, Sport and Competence" (*Music Analysis* 17/2 (1998): 182-94), explores this aspect of the genera theory.

rules of interpretation; it continues to show a strong predominance of Genus 4, with Genus 5 appearing a distant second. Note that, after reduction, Genus 5 also accounts for just two members of the set-class inventory, set classes 4-1 and 6-Z11. This example shows that the Squo reckons as equal the generic affiliation of all set classes, a circumstance that may produce unexpected results.

Table 2. Reduced Matrix and Squos of a Sample Case

	G4	G5
4-1		o
6-Z11		o
6-15	o	
6-21	o	
6-22	o	
6-31	o	
6-34	o	
Counts:	5	2

Squos in Descending Order:

G4:	.340
G5:	.246

One might bring the results in line with expectation, however, by recalibrating the index. To that end I propose the *exclusivity index* (*EI*), which ranks set classes according to the number of genera memberships they hold. The exclusivity index assigned to each set class correlates in inverse proportion to the generic exclusivity of that set class, and is expressed thus:

$$EI(x) = ((12 - y) / 11)$$

where x denotes a given set class, and y the number of genus memberships held by x.[11] The difference of $(12 - y)$ is divided by 11 to yield an EI in the range of 0 to 1.[12] The EI models the degree to which a set class can engage a particular genus: the fewer genera memberships a set class holds, the greater its power to engage a particular genus; the more memberships it holds, the lesser its power to engage one specific genus.

Set class 4-1, for instance, possesses the highest possible degree of exclusivity, because it holds membership in only one genus (Genus 5). Consequently, its capacity to engage a particular genus is accorded the highest EI: 1. Set class 6-22, on the other hand, bears the lowest possible degree of exclusivity, because it holds memberships in all genera but one; consequently, its power for generic engagement receives the lowest numerical value: .0909. The impact of set classes 4-1 and 6-22 on the Squo will now differ markedly. The former will strongly engage its genus while the latter will scarcely affect the status of any one of its eleven genera. Their EIs reflect the fact that set class 4-1 is significantly more representative of Genus 5 than set class 6-22 is of Genus 2, for example, or Genus 6, or Genus 9. Table 3 presents the EIs of trichords, tetrachords, pentachords, and hexachords.[13]

The ranking of set classes by the EI expression has several implications for assessing relations between genera and set-class inventories that will be touched upon in due course. The integration of the EI and the Squo is straightforward: values assumed by the variable terms in the Squo expression now reflect

[11]The EI assumes that each set class appears at least in one genus. Since set class 3-6 does not appear directly in any of the genera (i.e., the genus it generates is a subset of the genus produced by set class 3-8, Genus 2, and therefore it is eliminated), it is given the same EI as set class 3-8: 1.000.

[12]EIs are rounded off to four decimal places. In cases where the fifth place unit is 5, the fourth place unit is rounded to an even integer to avoid cumulative rounding errors.

[13]Because Forte's theory recognizes the symmetry of the twelve-pitch-class universe, it represents the larger set classes (heptachords through nonachords) by their smaller complements, or, in the case of hexachords, by their Z-related set classes.

Table 3. Exclusivity Indexes

Trichords:

1: 1.0000	2: .8182	3: .8182	4: .9091	5: 1.0000	6: 1.0000
7: .8182	8: 1.0000	9: 1.0000	10: 1.0000	11: .8182	12: 1.0000

Tetrachords:

1: 1.0000	2: . 9091	3: 1.0000	4: 1.0000	5: .9091	6: 1.0000
7: 1.0000	8: 1.0000	9: 1.0000	10: 1.0000	11: 1.0000	12: .8182
13: .8182	14: 1.0000	15: .9091	16: .9091	17: 1.0000	18: .8182
19: .7273	20: 1.0000	21: 1.0000	22: .9091	23: 1.0000	24: .9091
25: 1.0000	26: 1.0000	27: .8182	28: 1.0000	29: .9091	

Pentachords:

1: .9091	2: .7273	3: .7273	4: .4545	5: .7273	6: .8182
7: .9091	8: .7273	9: .6364	10: .6364	11: .4545	12: .8182
13: .3636	14: .7273	15: .9091	16: .6364	17: .6364	18: .4545
19: .6364	20: .8182	21: .7273	22: .5455	23: .7273	24: .6364
25: .6364	26: .2727	27: .7273	28: .6364	29: .4545	30: .3636
31: .4545	32: .6364	33: .9091	34: .7273	35: .9091	36: .3636
37: .6364	38: .4545				

Hexachords:

1: .7273	2: .4545	3: .4545	4: .6364	5: .2727	6: .9091
7: .9091	8: .3636	9: .2727	10: .1818	11: .0909	12: .3636
13: .5455	14: .2727	15: .0909	16: .1818	17: .3636	18: .2727
19: .4545	20: .7273	21: .0909	22: .0909	23: .5455	24: .1818
25: .4545	26: .6364	27: .4545	28: .5455	29: .5455	30: .4545
31: .0909	32: .7273	33: .4545	34: .0909	35: .9091	

set class rankings. To distinguish this altered status quotient from Forte's Squo, I will term it the Absolute Status Quotient (or *ASquo*).[14] The expression is now as follows:

$$\text{ASquo}(GA) = ((X / Y) / Z) \times 10$$

where each variable denotes what it did earlier, except that each of the three factors that contribute to the expression—the number of intersecting set classes, set-class inventory size, and genus size—sum the EI values of their set classes rather than simply the set classes themselves. Thus, each of the variable terms in the expression becomes *weighted*. Table 4 lists the sums of the EI values of Genera 1 through 12, the values assumed by *Z*.

Table 4. EIs of Genera 1 through 12

G1:	35.0909	G2:	36.8182	G3:	22.2727	G4:	10.0909
G5:	14.3636	G6:	21.2727	G7:	21.9091	G8:	19.1818
G9:	18.1818	G10:	19.1818	G11:	14.3636	G12:	21.2727

A simple example will both demonstrate how the ASquo is calculated and adumbrate its implications. Consider again the inventory of our earlier example, set class 4-1 and the gregarious hexachords. Singleton 4-1 has an EI value of 1 while each of the six hexachords has an EI value of .0909. The impact this weighting can already be seen in Table 5, where the weighted sums of intersecting set classes appear in the bottom row of the complete matrix.[15] Genus 5, which alone accounts for set class 4-1, now produces a greater sum than the other genera. (Notice in the matrix that the relevant EIs are displayed in the second

[14] I borrow the term "absolute" from Morris, "A Similarity Index for Pitch-Class Sets." His absolute similarity index (or *ASIM*) provides a meaningful way of comparing similarity indexes from set classes of any size; the EI achieves a weighting that is rather like the ASIM.

[15] Remember that the values of term *Z* in the ASquo are taken from Table 4.

Table 5. Complete Matrix and ASquos of a Sample Case

	[EIs]	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
4-1	[1.000]					o							
6-Z11	[.0909]	o	o	o		o	o	o	o	o	o	o	o
6-15	[.0909]	o	o	o	o	o	o	o	o	o	o		o
6-21	[.0909]	o	o	o	o	o	o	o	o	o	o		o
6-22	[.0909]	o	o		o	o	o	o	o	o	o	o	o
6-31	[.0909]	o	o	o	o		o	o	o	o	o	o	o
6-34	[.0909]	o	o	o	o		o	o	o	o	o	o	o

EI sums:

G1: .5455	G2: .5455	G3: .4545	G4: .4545
G5: 1.3636	G6: .5455	G7: .5455	G8: .5455
G9: .5455	G10: .3455	G11: .3636	G12: .5455

Set-Class Inventory EI Sum: 1.5455

ASquos in Descending Order:

G5:	.614
G4:	.291
G9:	.194
G8:	.184
G10:	.184
G6:	.166
G12:	.166
G11:	.164
G7:	.161
G3:	.132
G1:	.101
G2:	.096

column, next to the set-class inventory.) The ASquo of Genus 4, for instance, is calculated as follows:

$X =$	.4545	(sum of EIs of G4 set classes intersecting the SC inventory)
$Y =$	1.5455	(sum of EIs of the set-class inventory)
$Z =$	10.0909	(sum of EIs of members of Genus 4)

ASquo(4) = ((.4545 / 1.5455) / 10.0909) × 10 = .291 (rounded to 3 decimal places)

The matrix in Table 6, reduced according to Forte's five interpretive rules, provides a substantially different view of the generic profile of the sample set-class inventory. Weighting set classes according to their level of generic affiliation ensures that set class 4-1 will wield a greater influence in the interpretation of genus precedence than any of the six hexachords, and thus chromatic Genus 5 assumes the highest rank. Moreover, the interpretive rules attribute most of the set-class inventory to Genus 5: not only does it represent set class 4-1, but it also accounts for four of the six hexachords. Genus 5, then, is not only the highest ranked, but it is also the best represented. Conversely, the weighting of set classes greatly attenuates the influence of members that cross many generic boundaries, as we see in the case of the genera that intersect only the hexachords.

Table 6. Reduced Matrix and ASquos of a Sample Case

	G4	G5
4-1		o
6-Z11		o
6-15		o
6-21		o
6-22		o
6-31	o	
6-34	o	

ASquos in Descending Order:

G5:	.614
G4:	.291

To see the effect of the EI on the analysis of actual compositions, let us turn to two analyses Forte presents in his genera article. Table 7 provides the matrix from Webern's *Fünf Stücke für Orchester,* op. 10, no. 5 and includes results from both

Table 7. Complete Matrix, Webern, op. 10, no. 5

	[EIs]	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
4-3	[1.000]						o						
4-8	[1.000]	o											
4-9	[1.000]	o											
4-12	[.8182]		o	o			o						
4-Z15	[.9091]	o	o										
4-17	[1.000]									o			
4-18	[.8182]	o		o						o			
4-19	[.7273]				o				o	o	o		
4-Z29	[.9091]	o	o										
5-6	[.8182]	o	o						o				
5-7	[.9091]	o	o										
5-10	[.6364]	o	o	o			o	o					
5-16	[.6364]	o	o	o			o			o			
5-21	[.7273]				o				o	o	o		
5-23	[.7273]							o			o	o	o
5-32	[.6364]	o	o							o			o
5-35	[.9091]											o	o
5-Z38	[.4545]	o	o	o					o	o	o		o
6-14	[.2727]				o	o	o	o	o	o	o	o	o
6-15	[.0909]	o	o	o	o	o	o	o	o	o	o		o
6-16	[.1818]	o	o		o	o	o		o	o	o	o	o
6-Z19	[.4545]	o	o	o	o				o	o	o		
6-21	[.0909]	o	o	o	o	o	o	o	o	o	o		o
6-22	[.0909]	o	o		o	o	o	o	o	o	o	o	o
6-31	[.0909]	o	o	o	o		o	o	o	o	o	o	o

EI sums:

G1: 9.7273	G2: 7.7273	G3: 4.7273	G3: 2.7273
G5: .7273	G6: 3.9091	G7: 2.0000	G8: 4.0000
G9: 6.2727	G10: 3.9091	G11: 2.2727	G12: 3.5455

Set-Class Inventory EI Sum: 15.9091

Table 7. Complete Matrix, Webern, op. 10, no. 5 (cont.)

Indexes in Descending Order:

ASquos		Squos	
G9:	.217	G4:	.171
G1:	.174	G9:	.137
G4:	.170	G1:	.108
G3:	.133	G8:	.107
G2:	.132	G10:	.107
G8:	.131	G2:	.092
G10:	.128	G3:	.089
G6:	.116	G6:	.089
G12:	.105	G12:	.089
G11:	.100	G11:	.083
G7:	.057	G5:	.069
G5:	.032	G7:	.062

the weighted and unweighted indexes.[16] Genus 4, ranked first by the Squo, is surpassed in rank by both Genus 9 and Genus 1 using the ASquo. This restratification in the order of genus precedence suggests that many of the set classes in this inventory cross generic boundaries, and indeed this is so. Of the nine set classes of Genus 4 that intersect the inventory, six are hexachords that each retain membership in most of the other genera: the single most distinctive hexachord is 6-Z19, and it belongs to seven genera. While Genus 9 also intersects many gregarious set classes—in fact, it intersects all set classes of the inventory that belong to seven or more genera—it also contains numerous set classes whose degree of generic exclusivity ranges from moderate to high. The same holds true for Genus 1. When compared with those of Genera 9 and 1, the set classes of Genus 4 that intersect the set classes from the Webern piece are rather indistinct with respect to their generic affiliation.

When this matrix is reduced, following the five rules of interpretation, a contrasting generic profile emerges (see Table 8).

[16]Forte, "Pitch-Class Set Genera," 247-48. Because I calculated the Squos using the updated data from the errata list, their values in this Table differ slightly from those presented in Forte's analysis. Nevertheless, the order of precedence among the genera remains the same.

Table 8. Reduced Matrix, Webern, op. 10, no. 5

	G1	G2	G6	G9	G10	G11
4-3			o			
4-8	o					
4-9	o					
4-12		o				
4-Z15	o					
4-17				o		
4-18				o		
4-19				o		
4-Z29	o					
5-6	o					
5-7	o					
5-10	o					
5-16				o		
5-21				o		
5-23					o	
5-32				o		
5-35						o
5-Z38				o		
6-14				o		
6-15				o		
6-16				o		
6-Z19				o		
6-21				o		
6-22				o		
6-31				o		

Indexes in Descending Order:

ASquos:		Squos:	
G9:	.217	G4:	.171
G1:	.174	G9:	.137
G2:	.132	G1:	.108
G10:	.128	G10:	.107
G6:	.116	G2:	.092
G11:	.100	G6:	.089

Since the intersecting set classes of Genus 4 constitute a proper subset of the intersecting set classes of Genus 9, Genus 4 is eliminated entirely by virtue of the "Rule of Intersection." Under the ASquo, Genus 9 heads the list, and Genus 1 clearly becomes the secondary genus: it accounts for seven highly exclusive set classes while the remaining genera each account for a single set class.

A second example will suffice to demonstrate the range of effect the EI may have on generic analysis: Forte's analysis of Schoenberg's *Drei Klavierstücke,* op. 11, no. 1.[17] Tables 9 and 10 present the complete and reduced matrixes respectively. We find that the two indexes rank the genera in Table 9 almost identically, and in the interpreted matrix of Table 10, the rankings are identical, offering the same generic interpretation of set classes from the Schoenberg work. This suggests that many of the set classes in the leading genus are exclusive relative to those in the other genera that are well represented in the set-class inventory. When we compare the average EI of intersecting set classes of the leading genus[18] in Schoenberg's piece with the average EI of those in Webern's, the distinction becomes apparent.

In the Webern analysis, the average EI of the intersecting set classes of Genus 4 is .3030, whereas the average EI for those of Genus 9—which displaces Genus 4 in weighted ranking—is .4481. The intersecting set classes of Genus 9 are thus significantly more representative of their genus than are those in Genus 4, and it is this fact that brings about the reinterpretation of generic profile. In the Schoenberg analysis, on the other hand, the average EI of the intersecting set classes of Genus 8 is .5283, while the average EI for those of Genus 4 is .4773. Because the intersecting set classes of Genus 8 are, on the whole, more exclusive than those of Genus 4, the EI reinforces the existing

[17]Forte, "Pitch-Class Set Genera," 238-40.

[18]I am referring to the leading genus as determined by the unweighted Squo.

Table 9. Complete Matrix, Schoenberg, op. 11, no. 1

	[EIs]	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
3-3	[.8182]						o		o	o			
3-4	[.9091]								o		o		
4-7	[1.000]								o				
4-19	[.7273]				o				o	o	o		
5-13	[.3636]	o	o		o	o	o		o	o	o		
5-Z17	[.6364]				o		o		o	o	o		
5-Z18	[.4545]	o	o	o			o		o	o	o		
5-21	[.7273]				o				o	o	o		
5-Z37	[.6364]				o				o	o	o		o
5-Z38	[.4545]	o	o	o					o	o	o		o
6-Z3	[.4545]	o	o	o		o	o	o	o				
6-Z10	[.1818]	o	o	o		o	o	o	o	o	o		o
6-Z13	[.5454]	o	o	o			o	o		o			
6-16	[.1818]	o	o		o	o	o		o	o	o	o	o
6-Z19	[.4545]	o	o	o	o				o	o	o		
6-21	[.0909]	o	o	o	o	o	o	o	o	o	o		o
6-Z43	[.3636]	o	o	o			o		o	o	o		o

EI sums:

G1:3.5455	G2: 3.5455	G3:3.0000	G4: 3.8182
G5:1.2727	G6: 4.0909	G7:1.2727	G8: 8.4545
G9:6.6364	G10:6.1818	G11:0.1818	G12:1.9091

Set-class Inventory EI Sum: 9.0000

Indexes in Descending Order:

ASquos		Squos	
G8:	.490	G8:	.230
G4:	.420	G4:	.224
G9:	.406	G9:	.201
G10:	.358	G10:	.187
G6:	.214	G6:	.131
G3:	.150	G3:	.105
G1:	.112	G5:	.101
G2:	.107	G1:	.093
G12:	.100	G2:	.091
G5:	.099	G12:	.078
G7:	.065	G7:	.052
G11:	.014	G11:	.020

Table 10. Reduced Matrix, Schoenberg, op. 11, no. 1

	Asquo		Squo	
	G8	G9	G8	G9
3-3	o		o	
3-4	o		o	
4-7	o		o	
4-19	o		o	
5-13	o		o	
5-Z17	o		o	
5-Z18	o		o	
5-21	o		o	
5-Z37	o		o	
5-Z38	o		o	
6-Z3	o		o	
6-Z10	o		o	
6-Z13		o		o
6-16	o		o	
6-Z19	o		o	
6-21	o		o	
6-Z43	o		o	

Indexes in Descending Order:

ASquos	Squos
G8: .490	G8: .230
G9: .406	G9: .201

ranking. Where set classes contained in the leading genera are relatively exclusive, then, the unweighted Squo will model generic structure well; where these set classes are dispersed across many genera, the weighted ASquo affords a more discriminating perspective of generic structure.

Two further aspects of the EI invite brief comment. The first concerns EIs and generic size. The unweighted Squo does moderate disparities among generic sizes, of course. That is the function of term Z in the expression: it mitigates what otherwise would be the marked advantage of the larger genera in the calculation of the Squo. But the way in which it moderates these disparities takes into account only quantitative factors. Introducing weighted set classes into the equation does not minimize size differences among the genera—in fact, in the case of Genus 4, it accents the disparity[19] (see again Table 4)—but it places the disparities in the context of generic identity. Consider again the ASquos of our sample set-class inventory in Table 5. While the ASquos of most of the genera in Table 5 continue to be ranked in inverse proportion to their size (due primarily to the simplistic exclusivity ratios of most of the members of the set-class inventory), the index is sufficiently subtle to recognize that the *chromatic* rather than the *augmented* genus is more representative of our sample set-class inventory.

The second facet has to do with the EI's effect on our estimation of the exclusivity of each genus as a whole. Do some genera primarily comprise exclusive set classes while others comprise mostly set classes that cross generic boundaries? The arithmetic mean of the EI values of set classes that make up each genus offers a general sense of genus distinctiveness. (Table 11 lists the arithmetic means in descending order.) That Genera 1 and 2 head the list is somewhat surprising. Both are over a third larger than their nearest competitors, and one might at first assume that they would contain many gregarious set classes, which would result in a lower ranking. As it happens, though, the arithmetic means of their EIs show that, in addition to having the greatest number of set class members, they each hold proportionately more exclusive set classes than any of the smaller genera. With respect to the ASquo, this proportionately greater number of exclusive set classes in the two largest genera mitigates to some extent the disadvantage of their size.[20]

[19]When genera are represented by their EI sums (see again Table 4), the largest genus, Genus 2, has almost three and one half times the weight of the smallest genus, Genus 4; without weighting genus members—that is, by simply counting set classes—Genus 2 is just over three times the size of Genus 4.

[20]I include this point only as a matter of interest. There is no inherent benefit in regularizing the genera with respect to size, which, I believe, is an important aspect of generic uniqueness.

Table 11. Arithmetic Means of EI Sums of Each Genus

G2:	.5664
G1:	.5570
G11:	.4953
G5:	.4953
G3:	.4949
G7:	.4869
G4:	.4805
G6:	.4727
G12:	.4727
G8:	.4678
G10:	.4678
G9:	.4435

While my principal concern in this essay is the status quotient expression, for the sake of completeness I include a brief discussion of the EI's impact on the other of Forte's two indexes, the ***difference quotient*** (or *Difquo*), which compares genera with each other. While Forte sees this the *Difquo* as an essentially quantitative measure, he does draw interesting qualitative interpretations from it as well. The expression is as follows:[21]

$$\text{Difquo} = (X / Y) / 4$$

where X is the difference between the number of set classes of two genera or supragenera that do not intersect and those that do; Y is the difference between the number of set classes that make up the combined genera and the number of set classes they hold in common, thus factoring size into genus identity.[22] The Difquo measures commonalities among the genera themselves (or groups of genera that Forte labels "supragenera"). Its value may range

[21]Forte, "Pitch-Class Set Genera," 220-22.

[22]The quotient is divided by 4 to average the Difquos of each set class cardinality, three through six.

from -1, where the given genera are identical, to +1, where the intersection between given genera is nil.

As with the Squo, the Difquo does not differentiate between set classes that are widely dispersed among genera and ones that represent a single genus or a small number of genera. The Difquo depends only on a count of set class members. The EI offers another perspective on abstract generic relations by introducing a qualitative factor: as in the ASquo, those set classes that are particularly indicative of their genus are given greater significance in determining what I shall call the *Absolute Difference Quotient* (to distinguish it from the unweighted index). The integration of EI and the Difquo yields a more discriminative index of abstract generic relations, and its more precise reflection of generic uniqueness strengthens the basis for comparison. The absolute difference quotients (or *ADifquos*) of all twelve genera appear in Table 12.

The weighting of the Difquo has little consequence. While the range of values produced by the ADifquo is slightly greater than that of the Difquo, connections posited among the genera by the latter index remain essentially unchanged. Even the duplication of values that the unweighted index produced remains almost the same under the ADifquo. It follows that, with respect to analytic application, the EI does not significantly revise the Difquo's contribution to specific analytic decisions; the index remains an abstract, albeit interesting, measure of generic relatedness.

Through weighting, I have argued, one can address the problem of set classes that cross generic boundaries. Behind my solution, of course, lies the assumption that greater precision in measurement yields greater exactness in modeling. While this is the point of the present paper, I must also note that Forte's solution—to develop strategies of interpretation—invites us to consider the broader question of how we apply the genera theory. Which set classes from the piece does one include? How does musical form and context shape generic analysis? What sorts

Table 12. ADifquo Comparisons

	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
G1	.02169	.35137	.92251	.80261	.62002	.55545	.72735	.61774	.72735	.80261	.62002
G2		.38863	.81956	.80366	.58245	.64885	.75110	.70049	.75110	.80366	.58245
G3			.92651	.84516	.50460	.45526	.78657	.49682	.78657	.84516	.50460
G4				.92881	.90228	.95597	.56507	.54354	.56507	.92881	.90228
G5					.15903	.52550	.57291	.86466	.83408	.85424	.87503
G6						.38124	.42152	.47222	.81440	.87503	.71515
G7							.74438	.75264	.74438	.52550	.38124
G8								.32369	.29997	.83408	.81440
G9									.32369	.86466	.47222
G10										.57291	.42152
G11											.15903

of analytic statements can we draw from the genera matrix?[23] The answers to these and other questions will be shaped not only by a close examination of the theory itself but also by the repertoire to which it is applied.

[23]Some of these issues were touched upon in a symposium devoted to the genera theories of Allen Forte and Richard Parks at the Cambridge University Music Analysis Conference in 1997. Papers by Craig Ayrey, John Doerksen, Jonathan Dunsby, Christian Kennett, and Richard Parks, as well as a response by Allen Forte, offered wide-ranging perspectives on generic analysis, and were subsequently published in *Music Analysis* 17/2 (1998). Two of these papers treat topics that are closely allied to that of the present article. Christian Kennett's "Take Me Out to the Analysis Conference" (cited earlier) examines the statistical imbalance of genus size and its impact on generic analysis. John Doerksen's "Set-Class Salience and Forte's Theory of Genera," *Music Analysis* 17/2 (1998): 195-205, suggests a ranking scheme whereby compositionally prominent set classes are given greater recognition by the genera theory than are peripheral set classes.