Serialism and Neo-Riemannian Theory: Transformations and Hexatonic Cycles in Schoenberg's Modern Psalm op. 50c

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In Schoenberg's final and unfinished composition, *Modern Psalm* op. 50c, the particular ordering of the primary 6-20 <4,3,0,8,e,7> hexachord and its transformations governs a hierarchic and transformational cyclic structure. The circuitous and cyclic nature of the prime row of *Modern Psalm,* his final large-scale work, compelled Schoenberg to write a short essay on the source hexachord and its transformations entitled *Die Wunder-Reihe*.[1] The present paper explores the cyclic aspects of this ordered source set, examines unordered forms of the 6-20 [014589] hexachord and its trichordal subsets, and constructs cyclic *Tonnetz* spaces in a modified version of Richard Cohn's neo-Riemannian based hyper-hexatonic system.[2] Building upon Cohn's (1996) hyper-hexatonic system, especially aspects of tripartite divisions of the octave, Michael Siciliano's (2005) foray into the atonal repertoire of Schoenberg through toggling, and Joseph Straus's (2011) contextual-inversion spaces, this paper aims to extend and develop a theoretical methodology based on *Tonnetz* cycles in order to explore serial compositions.[3] Through an examination of the 6-20

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[1] Schoenberg 1977, 114.

[2] The traditional *Tonnetz*, or "tone-network," is a two-dimensional graph in which the axes represent the three intervals of a trichord (traditionally major and minor chords). Cohn's (1996) hexatonic system includes four co-cycles that each contains a form of the 6-20 hexachord. These co-cycles are ultimately fused to form a hyper-hexatonic system, a system that Cohn uses in order to explore hexatonic passages containing maximally smooth voice-leading cycles based on the consonant 3-11 trichord. In this study I will be focusing on equal divisions of the octave rather than voice-leading parsimony.

[3] There have been numerous extensions of neo-Riemannian theory and its application to post-tonal repertoire. Some notable contributions to this field of study include Clifton Callender (1998), Guy Capuzzo (2004), Dmitri Tymoczko (2008), and Edward Gollin (2011). Pertinent extensions include Siciliano (2005), who generates Cohn's (1996) hyper-hexatonic system through toggling cycles of set classes 3–3 and 3–4, Robert Morris (1988), who has suggested that it is

hexachord and its asymmetrical trichordal subsets, and utilizing modified versions of the familiar neo-Riemannian operators R, P, and L in addition to a newly-formed operator H (for *Halbton*), I generate the hyper-hexatonic system through *Tonnetz* cycles built upon set-class consistent series of 3–3 [014], 3–4 [015], or 3–11 [037] trichords and their transformations.[4] In Schoenberg's *Modern Psalm* op. 50c, these small- and large-scale *Tonnetz* cycles reveal the underlying pitch structure of the four component sections of the work. Using *Tonnetz* cycles, this study explores the compositional process of exhausting particular forms of a set class as a form-defining compositional construct.[5] The goal of this new theoretic model is to demonstrate how Schoenberg's small- and large-scale pitch design elegantly connects with the narrative design of the piece, especially within *Modern Psalm*. The large-scale form-defining cycles within *Modern Psalm* are tripartite and symmetrical constructs that ultimately represent the omnipresence of Schoenberg's God throughout the work, even when the narrator questions his or her relationship with God.[6]

This paper is divided into two sections. In the first section I will describe the abstract theoretical concepts including *Tonnetze* based on the asymmetrical trichordal subsets of the 6-20

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possible both to extend LPR and L'P'R' contextual transformations on all twelve trichordal set classes and also to generate *Tonnetze* for all twelve set classes, and Straus (2011), who extends neo-Riemannian operations to encompass all trichords and tetrachords (and is customized for larger sets), notably through RI chains.

4 In serial music, unlike tonal or post-tonal music, the order of pcs and subset types usually remains consistent throughout the composition eliminating the sometimes problematic aspect of applying neo-Riemannian theory to atonal and tonal repertoire, where pitch collections are variable.

5 Within the opening four measures of Berg's "Schlafend tragt man mich" op. 2, all six forms of set class 4–25(0268) are stated prior to a restatement of the 4–25 tetrachord with which the cycle began. The restatement of this trichord coincides with the last syllable of "Heimatland," which captures the "idea expressed in the text of returning to a homeland" (Straus 2005, 127).

6 In particular, the trichord <4, 0, 8>, which is a symmetrical and tripartite structure, is utilized by Schoenberg both in *Modern Psalm* and in *A Survivor from Warsaw,* a work that was written four years prior to *Modern Psalm*. The tripartite pitch design in *A Survivor from Warsaw* ultimately represents the omnipresence of Schoenberg's ineffable God throughout the dark narrative of the work (Argentino, 2013). The final section will discuss this further.

hexachord, adapted versions of R, P, L, and H transformations, and various *Tonnetz* cycles which are utilized to navigate through an altered version of Cohn's hyper-hexatonic system. I then illustrate how these theoretical constructs contribute to a short analysis of a passage from Schoenberg's "Nacht" drawn from *Pierrot Lunaire*. The second section includes an examination of the text and source set for *Modern Psalm*, followed by an in-depth analysis of the work, fusing the theoretical constructs with the text.

Part I

1. The Hexatonic Hexachord

There are only four unique forms of the 6–20 hexachord, also commonly referred to as the hexatonic hexachord, Babbitt's "third order" source set.[7] The high degree of symmetry in the 6–20 hexachord distinguishes it from other hexachords, as it contains only thirteen subsets. What the 6–20 hexachord lacks in subsets is made up for by the numerous ways each subset can be obtained. Example 1 uses formatting to indicate the number of times a given subset is embedded within the hexachord: bold indicates six appearances, italics shows those with three forms, and the remaining 3–12 trichord appears in the hexachord twice. With the exclusion of the 3–12 [048] trichord, which cannot form an aggregate-inclusive *Tonnetz* space, it is possible to construct *Tonnetz* spaces for trichords 3–3, 3–4, and 3–11 that contain all twelve pcs hereafter called the aggregate.[8]

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[7] Babbitt 1955, 57-58. Like other symmetric collections such as the chromatic, whole-tone, and octatonic, one of the hexatonic collection's defining characteristics is the repetition of a series of intervallic subunits within the aggregate, i.e., the alternation of intervals 1 and 3.

[8] In order for a trichord's *Tonnetz* to contain all twelve pcs, the trichord must contain either interval 1 or interval 5 (or their respective inversions 11 and 7), as both these intervals take twelve moves to cycle through the aggregate without repeating any notes. The only trichords that do not contain these intervals, and thus cannot form aggregate-inclusive *Tonnetz* spaces, are trichords 3–6[026], 3–8[046], 3–12[048] (all of which are embedded members of the whole-tone collection), and 3–10[036] (a member of the octatonic collection).

Example 1. Subsets of 6-20[014589]

2-note	*2-1[01]*	*2-3[03]*	**2-4[04]**	*2-5[05]*
3-note	**3-3[014]**	**3-4[015]**	**3-11[037]**	<u>3-12[048]</u>
4-note	*4-7[0145]*	*4-17[0347]*	**4-19[0148]**	*4-20[0158]*
5-note	**5-21[01458]**			

(**Bold=6 forms**)*(Italicized=3 forms)*(<u>*3-12=2 forms*</u>)

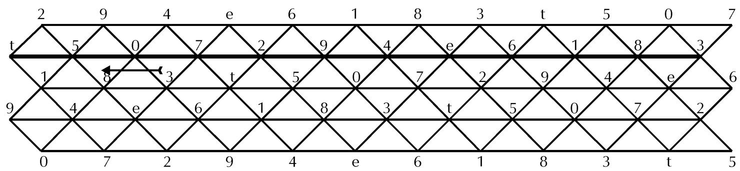
Examples 2.a, 3.a, and 4.a show the *Tonnetze* and cycles for each of the trichordal subsets of the 6–20 hexachord (except for the 3–12[048] case). Example 2.a shows the *Tonnetz* for trichord 3–11, the equal-tempered neo-Riemannian *Tonnetz* with integers; Example 3.a shows the 3–4 trichord *Tonnetz* space; and Example 4.a shows the 3–3 *Tonnetz* that shares the same topographic space as *Tonnetz* 3–11. Each *Tonnetz* space features three axes; each axis individually represents a series of one of the three component intervals (i.e., interval classes) of each trichord, and collectively the intersection of the three axes—which form a triangle—represents one of the possible forms (i.e., prime or inverted) of each trichord. Analogous to the neo-Riemannian 3–11 *Tonnetz* shown in Example 2.a, the 3–4 and 3–3 trichords' inversions shown in Examples 3.a and 4.a are also visually represented by the opposing apexes of adjacent triangles.

Example 5 contains adaptations of the familiar neo-Riemannian transformations R, P, L, as well as a newly formed transformation entitled H, that individually express the relationship between inversionally-related forms of trichords in each *Tonnetz* space.[9] Borrowing the symmetrical attributes of the neo-Riemannian transformations R, P, and L, as reflecting pitch classes

[9] Cohn (1996, 1997, 1998, 1999) exhaustively studied the voice-leading smoothness of P, L, and R along with their *Tonnetz* mappings. The *Tonnetz* appears in Riemann (1873), but diagrams of a similar nature appeared in Euler's work (1739) a century before (Mooney 1996, 1–2).

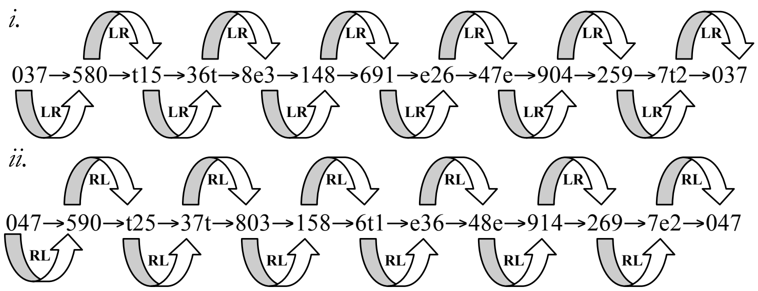
Example 2. 3-11 Tonnetz *space and* Tonnetz *axis cycles*

a) *Tonnetz* 3–11[037](T_3, T_4, T_5)



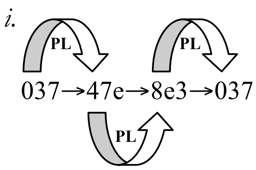
b) 3-11 (T_5) Tonnetz *axis cycles, length 12*

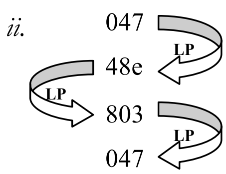
LR=T_5
RL=T_5



c) 3-11 (T_4) Tonnetz *axis cycles, length 3*

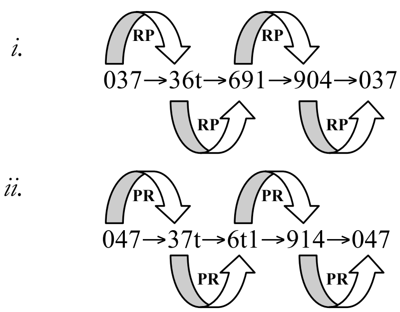
PL=T_4
LP=T_4





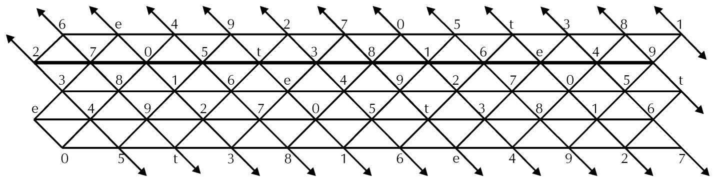
d) 3-11 (T_3) Tonnetz *axis cycles, length 4*

PL=T_4
LP=T_4



Example 3. 3-4 Tonnetz *space and* Tonnetz *axis cycles*

a) Tonnetz *3-4[015](T₁, T₄, T₅)*



b) 3-4(T_5) Tonnetz *axis cycles, length 12*

RH=T_5
HR=T_5

i.

015→ 56t→te3→348→891→126→67e→e04→459→9t2→237→780→015

ii.

045→59t→t23→378→801→156→6te→e34→489→912→267→7e0→045

c) 3-4(T_4) Tonnetz *axis cycles, length 3*

PH=T_4
HP=T_4

i.

015→459→891→015

ii.

045
489
801
045

d) 3-4(T_1) Tonnetz *axis cycles, length 12*

RP=T_1
PR=T_1

i.

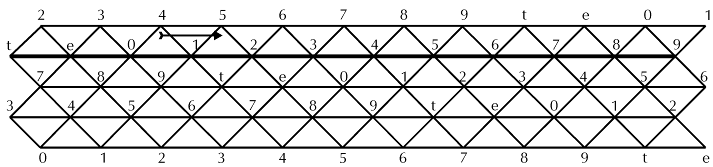
015→126→237→348→459→56t→67e →780→891→9t2→te3→e04→015

ii.

045→156→267→378→489→59t→6te→7e0→801→912→t23→e34→045

Example 4. 3-3 Tonnetz *space and* Tonnetz *axis cycles*

a) Tonnetz *3-3[014](T_1, T_3, T_4)*



b) 3-3(T_1) or <014>(T_1) Tonnetz *axis cycles, length 12*

$LR=T_1$
$RL=T_1$

i)

014→ 125→236→347→458→ 569→67t → 78e→890→9t1→ te2→e03→014

ii)

034→ 145→256→367→478→ 589→69t → 7te→8e0→901→ t12→e23→034

c) 3-3(T_4) Tonnetz *axis cycles, length 3*

$LH=T_4$
$HL=T_4$

i.

014→458→890→014

ii.

034 → 478 → 8e0 → 034

d) 3-3(T_3) Tonnetz *axis cycles, length 4*

$RH=T_3$
$HR=T_3$

i.

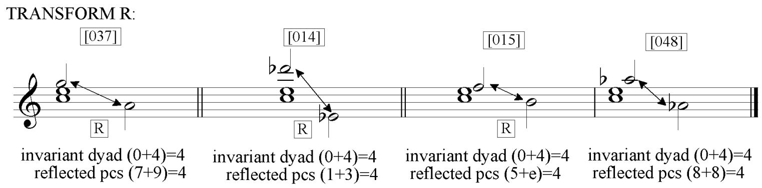
014→347→67t→9t1→014

ii.

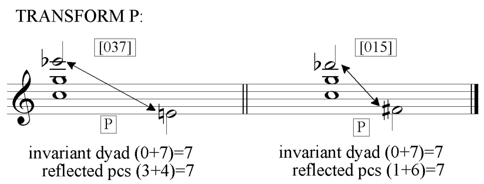
034→367→69t→901→034

Example 5. R, P, L, and H Transformations

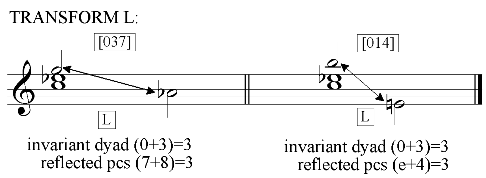
a)



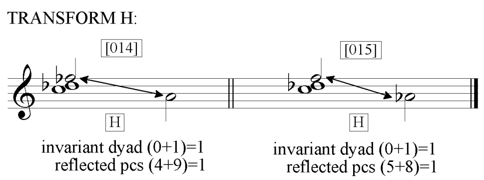
b)



c)



(d)



against a pair of stationary pcs of a 3–11 trichord, I will adapt R, P, L, and H in order to describe the relationships between inversionally-related forms of trichords 3–3, 3–4, 3–11, and 3–12. Transformations R, P, and L are interpreted here as transformations between corresponding trichordal sets that share an invariant dyad, respectively, an ic 4, ic 5, or ic 3 dyad, where the sum of the invariant dyad (*modulo* 12) will equal the sum of the reflected pcs (*modulo* 12).[10] Example 5.a shows transformation R

[10] Following Cohn (1997, 2-4), who provides a generalization of R, P, and L transformations for all twelve trichords, I will exclusively focus on the trichordal

operating on all four hexatonic trichords [037], [014], [015], and [048], as all four trichord types contain at least one component interval-class 4 dyad. R relates the first pair of 3–11 trichords in Example 5.a, {0,4,7}{0,4,9}, as the sum of the pcs of the invariant ic 4 dyad (i.e., 0+4=4), and the sum of the reflected pcs (i.e., 7+9=4) are equivalent. R also relates trichord pairs {0,1,4}{0,3,4,}, {0,4,5}{e,0,4}, and {0,4,8}{0,4,8}, as the sum of the invariant {0,4} dyad equals the sum of the reflected pcs for each inversionally-related trichord pair.[11] Example 5.b contains a pair of [037] trichords and a pair of [015] trichords that are related by transformation P, as these are the only hexatonic trichords that contain a component ic 5 dyad. As shown in Example 5.b, the sum of the invariant ic 5 dyad (i.e., 0+7) is equivalent to the sum of the reflected pcs for each pair of trichord types. Transformation L relates pairs of [037] trichords or pairs of [014] trichords, as these are the only hexatonic trichords that contain a component interval-class 3 dyad. In Example 5.c, the invariant dyads are equal to each pair of reflected pcs. Interval class 1 is the only hexatonic interval which is not represented by transformations P, R, and L. Transformation H will incorporate ic 1, and will also be analogous to the versions of R, P, and L described above—that is, trichords that are related by H will share an invariant interval-class 1 dyad, and the sum of the ic 1 dyad will equal the sum of the reflected pcs.[12] In Example 5.d, trichord pairs {0,1,4}{9,0,1}, and {0,1,5}{8,0,1}—respectively a pair of [014] and [015] trichords—are related by the newly formed H transformation, for the sum of the invariant ic 1 dyad (1+0=1) equals the sum of the reflected pcs. When transformations R, P, L, and H are combined in pairs, such as PR or PLPL, their transformations on trichords are equivalent to transposition operations.

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subsets of the 6–20 hexachord. My RPL and H transformations, akin to the familiar RPL transformations, are both interval-class and pitch-class specific.

[11] When triads contain two or more equivalent interval classes (i.e., [012], [027], [036], and [048]), any of the interval-class equivalent dyads can remain invariant.

[12] I restrict my consideration in this paper to trichords [037], [014] and [015]. See Straus (2011) for a comprehensive examination of all trichords related through contextual inversions.

2. Tonnetz Axis Cycles

The parentheses that follow each of the trichords in the headings of Examples 2.a, 3.a, and 4.a include the three possible transposition values for each cycle (excluding inversions) that are equivalent to the three types of axes within each *Tonnetz* space (excluding retrogressions).[13] Since the representative trichord in each of the three *Tonnetz* spaces is a subset of the 6-20 hexachord, which contains only four ics, there are only four possible interval cycles: a T_1-cycle, a T_3-cycle, a T_4-cycle, and a T_5-cycle (and their inversions). I will consider all T_n-interval cycles (*modulo* 12 inversion) identical under retrogression; that is, the two possible forms of any simple $T_{(n)}$-cycle that are identical under rotation will be considered equivalent.[14] A 014(T_4)-cycle, for example, is the retrogression of a 014(T_8)-cycle: the cycles differ only in direction. When it is not possible to determine the direction of a cycle (see Examples 2.c.ii, 3.c.ii, and 4.c.ii) the smaller T_n-value will be used as the default.[15]

In Examples 2.a, 3.a, and 4.a, the parallel axes within each *Tonnetz* consist of the same interval type from pc to pc (herein node to node), and the transposition of the representative trichord of each *Tonnetz* space along one of the three axes by the interval between adjacent nodes creates one of three possible cycle lengths: a 1-cycle (or a T_1-cycle) or a 5-cycle (or a T_5-cycle) has length 12 as shown in Example 3.b or 3.d, a 4-cycle (or a T_4-cycle) has length 3 as shown in Example 2.c, and a 3-cycle (or a T_3-cycle) has length 4 as shown in Example 2.d.[16] The transposition of any set-class consistent prime or inverted form of a trichord that traverses through all the nodes on an axis, thus exhausting either the prime

[13] The nomenclature for each *Tonnetz* axis cycle will include either a prime Forte name or ordered pcs followed by a T_n-value representing the interval cycle.

[14] For the remainder of the paper, the following interval cycles will be understood as equivalents: T_1 and T_e, T_3 and T_9, T_4 and T_8, and T_5 and T_7.

[15] For a concise description of simple interval cycles, compound interval cycles, double interval cycles, and triple interval cycles, see Gollin (2007).

[16] A cycle of length n is generated by $+k$ modulo n if and only if k is co-prime to n.

or inverted forms of the trichord along an axis, will be understood as a *Tonnetz* axis cycle.

The 3-3 *Tonnetz* space shown in Example 4 is analogous to the familiar neo-Riemannian *Tonnetz*. Example 4.b.i shows a 3–3(T_1) *Tonnetz* axis cycle, a cycle that is extracted from the east/west alley of Example 4.a, where the starting position of the cycle is demarcated with an arrow. The twelve unidirectional LR arrows of Example 4.b.i transform adjacent <0,1,4> trichords upwards by T_1. Throughout this 3–3(T_1) all twelve pc members of the interval-1 axis in the 3-3 *Tonnetz* are exhausted (in Example 4.a, see the heavy line/axis), and all possible spellings of the <0,1,4> forms of the 3–3 trichord are stated. Example 4.b.ii contains the R-related inverse of the <0,1,4> trichord of Example 4.b.i, where the twelve RL arrows exhaust all possible spellings of the <0,3,4> forms of the trichord.[17] Although I have included examples of length 3, length 4, and length 12 *Tonnetz* axis cycles, for the purposes of this paper I will focus on length 3 cycles as they are integral to the dodecaphonic examples I have drawn from Schoenberg's repertoire.

The (T_4)-*Tonnetz* axis cycle is the only cycle that occurs in all three *Tonnetze* spaces, as all of 6-20's trichordal subsets contain ic 4.[18] Although the 3–12 trichord cannot form an aggregate-inclusive *Tonnetz*, it has a special place as an embedded axis in the *Tonnetze* of set-class 6–20's other trichordal subsets: trichords 3–3, 3–4, and 3–11 can all project three-move (T_4)-cycles in the horizontal or vertical plane as shown in Examples 2.c, 3.c, and 4.c. In these examples the cycles of the three different trichords are shown in a melodic (horizontal) or chordal (vertical) dimension, with Examples 2.c.i, 3.c.i, and 4.c.i representing linear space, where the respective 3–11, 3–4, and 3–3(T_4)-*Tonnetz* axis cycles occur horizontally with the incipit notes of each cycle projecting

17 Since pairs of length-12 *Tonnetz* axis cycles contain all the possible trichordal members of the group and the complete aggregate, all other *Tonnetz* axis cycles of the same trichordal form are embedded with the cycle genus, including all length-3 and length-4 cycles.

18 Although length-12 *Tonnetz* axis cycles occur in all three *Tonnetz* spaces, they are predicated on two cycle types: (T_5) and (T_1). Neither of these cycle types occurs in all three *Tonnetz* spaces.

horizontal 3-12 trichords, versus Examples 2.c.ii, 3.c.ii, and 4.c.ii, where the 3–11, 3–4, and 3–3 (T_4)-*Tonnetz* axis cycles occur vertically, projecting harmonic 3–12 trichords.

In Example 2.c.i adjacent 3–11 trichords share an invariant pc, the unidirectional arrows transpose each trichord upwards by four semitones through the operation PL, dividing the aggregate into three symmetrical groups, and the cycle exhausts all the nodes of the interval-4 axis. Example 2.c.ii contains the P-related form of the 3–11 trichord of Example 2.c.i. The cycle in Example 2.c.ii has harmonically-stacked trichords in which all the horizontal trichords are 3–11s and all the vertical trichords are 3–12s. The 3–11(T_4) default nomenclature is used as the trichords in this cycle are assembled harmonically, and it is therefore not possible to determine the direction of the cycle.[19] With the exception of the undeterminable direction of any harmonic *Tonnetz* axis cycle, harmonic and linear cycles are identical. Thus in a manner similar to Example 2.c.i, adjacent trichords in Example 2.c.ii share an invariant dyad. Each trichord is related by interval 4, the cycle exhausts all the nodes of the interval-4 axis, and the aggregate is subdivided into three symmetrical groups by the *Tonnetz* axis cycle projecting 3–12 trichords. Example 3.c.ii and 4.c.ii correspond to Example 2.c.ii, with exception that the trichords in Examples 3.c.ii and 4.c.ii are respectively 3–4s and 3–3s.

Since every alley in trichordal *Tonnetz* spaces comprises two axes that share a prime and inverted form of the set, there can be a second type of cyclic axis exhaustion in which both axes of an alley are depleted. This process, contingent upon the particular musical surface, can be understood as either the union of two or more staggered *Tonnetz* axis cycles as shown in Example 6.a, or as the combination of two or more simultaneous *Tonnetz* axis cycles as shown in Example 6.b.[20]

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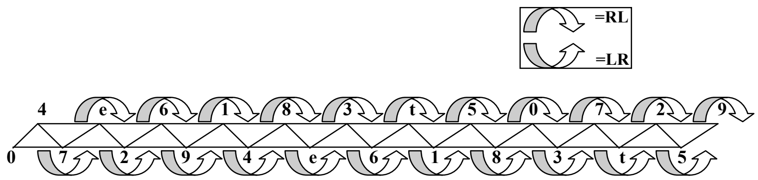
[19] Although Example 2.c.ii contains the unidirectional neo-Riemannian transformational LP arrows, PL arrows moving in the opposing direction are viable alternatives as the cycle's direction is ambiguous.

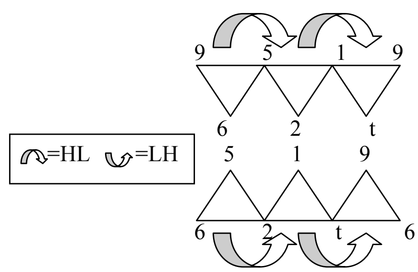
[20] The theoretical constructs and musical excerpts within this study can be perceived in multiple ways depending upon the musical surface of a work. For example, the cycle in Example 6.a can be understood as a simple R/L or RI chain (see Straus 2011), or, as I am suggesting, as two independent T-related cycles.

Example 6. Tonnetz *alley cycles*

a) Staggered union of two twelve-move 3-11(T_7) Tonnetz *axis cycles*



b) Two simultaneous 3-move 3-3(T_8) Tonnetz *axis cycles*



Example 6.a depicts two independent and symmetrically-opposed 3–11(T_7) *Tonnetz* axis cycles that deplete all twenty-four trichords in the W/E alley of *Tonnetz* 3–11. Example 6.b shows two simultaneous 3–3(T_8) cycles that deplete all six trichords of the SW/NE alley of *Tonnetz* 3–3. I define the cyclic depletion of all trichordal members of an alley as a "*Tonnetz* alley cycle."

The final *Tonnetz* cycle type that I will discuss, which I call an *invariant hexatonic alley cycle*, is formed through the union of two simultaneous length-3 *Tonnetz* axis cycles. It is a special type of *Tonnetz* alley cycle where the generating trichords of each simultaneous *Tonnetz* axis cycle are hexatonically polar-related; that is, the simultaneous and hexatonically complementary trichords do not share any commons pcs.[21] The union of each leg of these hexatonic polar-related length-3 *Tonnetz* axis cycles forms a 6–20 hexachord that remains invariant throughout the cycle.

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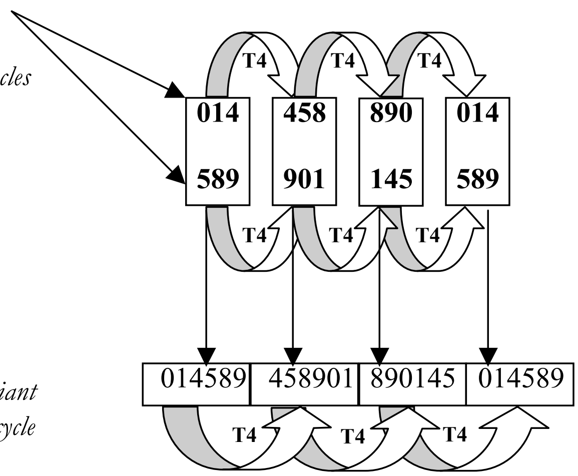
[21] The most efficient way of identifying a region entails presenting two triads that are in a hexatonic polar relation. See Cohn 1996, 19.

Example 7. Polar-related Tonnetz *alley cycles and an invariant hexatonic alley cycle*

a) Two 3-3(T_4) polar-related Tonnetz axis cycles

b) 6-20(T_4) invariant hexatonic alley cycle



Example 7.a shows two simultaneous and polar-related T_4-*Tonnetz* axis cycles that become the basis for the invariant hexatonic alley cycle shown in Example 7.b. Example 7.b shows the union of these two cycles, where each leg of the T_4 cycle contains the same invariant 6–20 hexachord. Although the internal ordering of the 6–20 hexachord changes throughout the series, the pc content remains invariant in each leg of the cycle. The incipit notes of these invariant hexatonic alley cycles can project larger-scale vertical or harmonic 3–12 trichords and will be labeled as either 6–20(T_4) or 6–20(T_8), depending on the direction of the cycle. Hexatonic regions and cycles are an integral aspect of Richard Cohn's hexatonic system, shown in Example 8.[22] Although Cohn

[22] Cohn demonstrates that consonant triads have the ability to form maximally smooth cycles, from which he constructs his hexatonic and hyper-hexatonic systems as shown in Example 8 of the present article (see Cohn 1996). Cohn designates each of the southwest/northeast alleys taken from *Tonnetz* 3–11 as a hexatonic system, represented by the four forms of the 6-20 hexachord labeled W (west), N (north), E (east) and S (south). Each of the co-cycles shares three pcs with each adjacent system (i.e., W and N share the trichord {0,4,8}), while opposing hexatonic systems are complementary. Sequential motion through any one of the co-cycles' trichords, represented here by the four *Tonnetz* alleys in

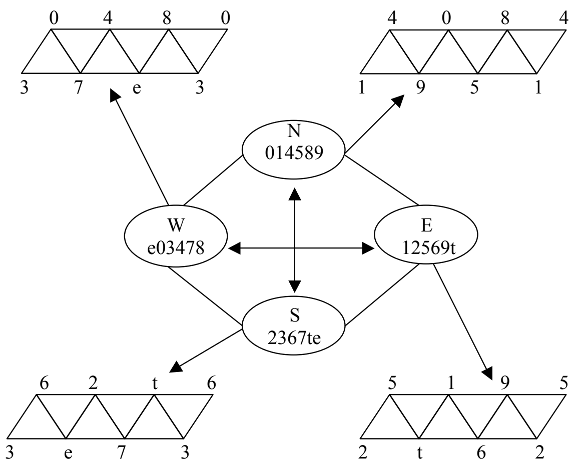
emphasizes both maximally smooth voice leading and equal subdivision of the octave, I utilize new definitions of R, P, L, and H while emphasizing only equal divisions of the octave, which I have described as *Tonnetz* axis cycles, *Tonnetz* alley cycles and invariant hexatonic alley cycles. Example 9 contains a modified version of Cohn's hyper-hexatonic system in which alleys from the 3–3 and 3–4 *Tonnetze* generate the same hexatonic space through cycles of 3–3 or 3–4 trichords. The 3-3 or 3-4 cycles can be generated through *Tonnetz* axis cycles, *Tonnetz* alley cycles, and invariant hexatonic alley cycles.

Example 10.a features a musical excerpt taken from the piano part of Schoenberg's "Nacht" from *Pierrot Lunaire*, which is a segment taken from an extended version of this cyclic chain (see m. 19 and mm. 21-23). This passage contains four 3–3(T_8)-*Tonnetz* axis cycles; their respective trichords demarcate two of the possible four forms of the 6–20 hexachord: H1 represents hexatonic region {1,2,5,6,9,t}, and H3 represents hexatonic region {e,0,3,4,7,8}. These spaces coincide with the Western and Eastern co-cycles shown in Example 9. The 3–3 trichord plays an integral motivic transformational role throughout "Nacht.[23] On the surface, the pairs of voices contain consecutive 3–3 trichords that alternate between H1 and H3, and each note of these trichords supports two transformational ic 4-related descending 6–35 [02468t] whole tone collections in the upper and lower registers. Note that in Example 10.a, every H1 region is comprised of a pair of 3–3 trichords that are polar-related, forming an invariant hexatonic alley cycle as shown in Example 8.b, while the H3 cycle, containing only 4 of the possible 6 pcs of the region, forms a *Tonnetz* alley cycle as shown in Example 8.c. These two cycles move through all the possible spellings of the 3–3 trichords embedded within H1 and H3 (i.e., all the 3-3 trichords in the modified version of Cohn's Western and Eastern regions of his hyper-hexatonic system).

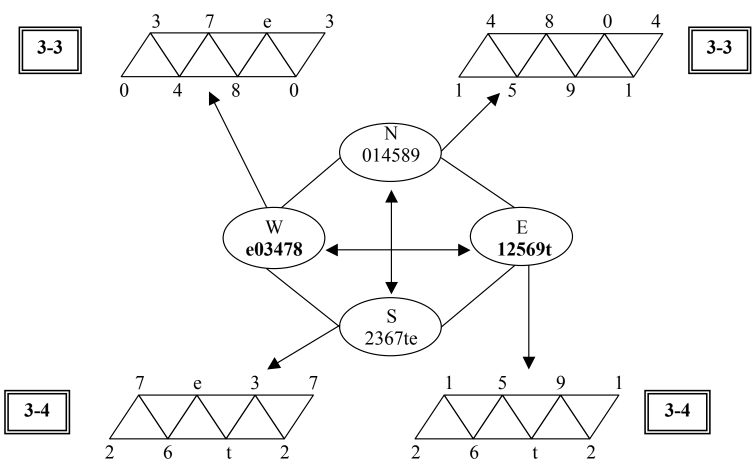
Example 8, results in what Cohn describes as a maximally smooth cycle: all the trichords belong to the same set class, adjacent trichords share an invariant dyad plus a pair of pcs a semitone apart, and the cycle forms a group.

[23] See Jeffrey Gillespie (1992) for a comprehensive analysis of 3–3 transformations throughout "Nacht."

Example 8. Cohn's four hexatonic systems with 3-11[037] Tonnetz *alleys and Cohn's hyper-hexatonic system*



Example 9. The hyper-hexatonic system with Tonnetz *alleys from the 3-3 and 3-4* Tonnetze

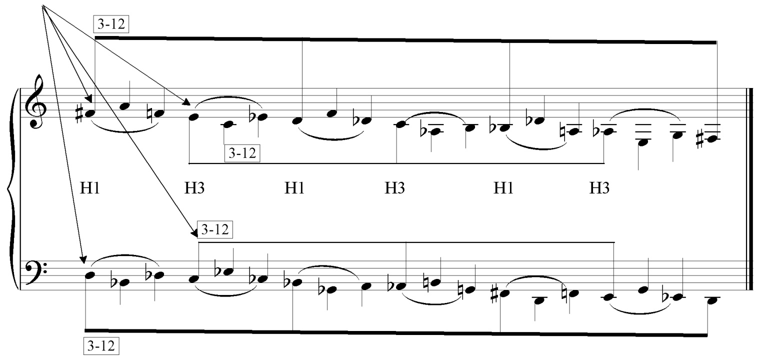


Example 10. Schoenberg's "Nacht," Hexatonic alley cycles and Tonnetz alley cycle

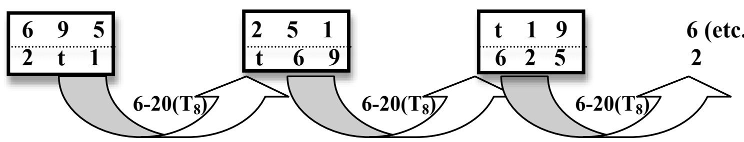
a) Piano part from Schoenberg's "Nacht" from Pierrot Lunaire *(mm. 21–2)*

H1=hexatonic region {12569t}, H3=hexatonic region {e03478}

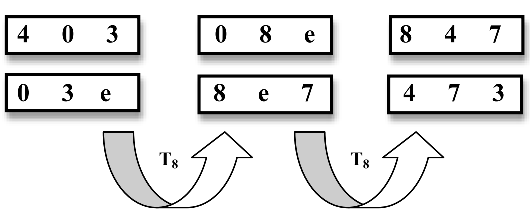
Four 3-3(T_8)-Tonnetz axis cycles



b) H1: 6-20 (T_8) Hexatonic alley cycle shown with pcs



c) H3: Tonnetz *alley cycle (comprised of two 3-3 (T_8)* Tonnetz *axis cycles)*



At a deeper level supported by rhythmic placement (the first of every six pitches is on the beat), the passage reveals a tripartite (3–12) division of the octave. The "Nacht" passage begins with the following text: *Und vom Himmel erdenwärts Senken sich mit schweren Schwingen* (and from heaven toward the earth, sinking down on heavy pinions). The cycles commence on the word *Senken* immediately following the narrator's reference to "heaven," highlighting the fact that this passage, which literally descends from a high to a low range, bridges the descent from "heaven" back "toward the earth." For Schoenberg, tripartite symmetrical cycles have an extra-musical meaning, as they often represent or are associated with the divine.[24]

The "Nacht" analysis, albeit based upon a brief excerpt, showcases paradigms that are often associated with hexatonic passages, including tripartite divisions of the aggregate, regional set class depletion of particular forms, *Tonnetz* cycles, transformational 6–35 collections, and in Schoenberg's case, divinity. All of these associations also occur as form-defining elements throughout Schoenberg's *Modern Psalm*, where he utilizes these hexatonic paradigms and their extra-musical associations with the divine over large-scale sections of the composition.

[24] Although tripartite structures are often associated with Christianity and the Holy Trinity, this particular augmented triad, which has been described as the "God motif" and the "perfection and immutability of the Jewish-God idea," is also symmetrical, an attribute that is associated with perfection (Schiller 2003, 103-4; Jackson 1997, 283). As stated in Argentino 2013 (12-13, fn. 6): "Throughout his career Schoenberg struggled with musically representing God, and there has been considerable discussion regarding symmetry as an expression of perfection. For example, in Schoenberg's *Moses und Aron*, God, who is ineffable within the Jewish tradition, is manifested indirectly throughout the work by other divine emanations. Authors such as David Lewin (1967) and Michael Cherlin (2007) have demonstrated that relationships between the rows that are not easily labelled or named indirectly represent that which is unrepresentable throughout Schoenberg's work: God."

PART II: *Modern Psalm*

Modern Psalm is set for a six-voice chorus, speaker, and orchestra, and the music remained incomplete at the time of Schoenberg's death. Schoenberg wrote a series of Psalm texts between September 1950 and July 1951 and set music to a portion of the first completed Psalm text, which became *Modern Psalm,* op. 50c. Example 11 contains the text, sectional divisions, and the schema of the text. The formal layout of the piece is neatly divided by three fermatas into four large sections numbered 1-4, as shown in the first column of Example 11. The first section is the only part that combines both speaker and six-part choir; afterwards the four-part choir and speaker alternate as shown in column one. As noted by Robert Sprecht and Thomas Couvillon, Schoenberg's original manuscript clearly distributes the nine sentences of *Der Erste Psalm* into seven paragraphs as shown with ordinal numbers that I have added to Mark Risinger's English translation shown in Example 12.[25] The text schema, shown in Example 12.b with capital letters, articulates the thematic content of the text: section A/A' consists of paragraphs one and seven, section B/B' consists of paragraphs two, three, five, and six, and section C contains the fourth paragraph. There is a thematic parallelism found within the seven paragraphs of the *Psalm*: paragraphs one, four, and seven are primarily about God, and paragraphs two, three, five, and six are primarily about the speaker's relationship with God. Paragraphs one and seven are closely linked through their point of view, as they are the only paragraphs written in the second person (all other paragraphs are written in the first person), and they both commence with the phrase: "*O du mein Gott*." Paragraphs two and three correspond to paragraphs five and six, and focus on the speaker's exploration of his or her own relationship with God; the narrator in paragraphs two and three humbly questions the efficacy of his or her own prayer, and in paragraphs five and six he or she ultimately decides to pray nonetheless.

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[25] See Couvillon (2002, 65); Sprecht (1976, 392); and Risinger (2000, 291-92). Hereafter *Psalm* will refer to the manuscript text and *Modern Psalm* will refer to op. 50c. The paragraph distribution is not clear in the typeset version of the text, but is clear in the handwritten version. See Schoenberg (1956).

Example 11. Modern Psalm *Text*

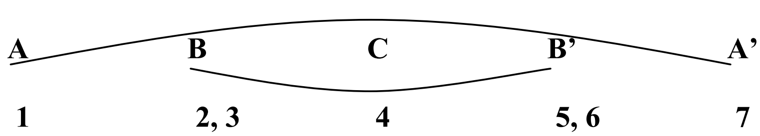
Section	**Text**	**Text Scheme**
1. mm. 1-32 Choir & Speaker	O, du mein Gott:	A
	alle Völker preisen dich	
	und versichern dich ihrer Ergebenheit.	
	Was aber kann es dir bedeuten, ob ich das auch tue oder nicht?	B
	Wer bin ich, (dass ich glauben soll, mein Gebet sei eine Notwendigkeit?)[note: parenthetical material is not repeated.]	
Speaker	Wenn ich Gott sage, weiss ich, dass ich damit von dem Einzigen, Ewigen, Allmächtigen, All-wissenden und Unvorstellbaren spreche, von dem ichmir ein Bild weder machen kann noch soll. An den ich keinen Anspruch erheben darf oder kann, der mein heissestes Gebet erfüllen oder nicht beachten wird.	C
2. mm. 33-50 Choir	Wenn ich Gott sage, weiss ich, dass ich damit von dem	C'
	Einzigen,	
	Ewigen, Allmächtigen, All-wissenden und	
	Unvorstellbaren spreche	
	der mein heissestes Gebet erfüllen oder nicht beachten wird	
3. mm. 51-72 Speaker	Und trotzdem bete ich, wie alles Lebende betet; trotzdem erbitte ich Gnaden und Wunder; Erfüllungen. Trotzdem bete ich, denn ich will nicht des beseligenden Gefüls der Einigkeit, der Verbindung mit dir, verlustig werden.	B
4. mm. 72-86 Choir	Und trotzdem bete ich, wie alles Lebende betet;	B'
	trotzdem erbitte ich Gnaden und Wunder;	
	Erfüllungen.	
	Und trotzdem bete ich	

Example 12. Modern Psalm *Sentence Distribution and Text Schema*

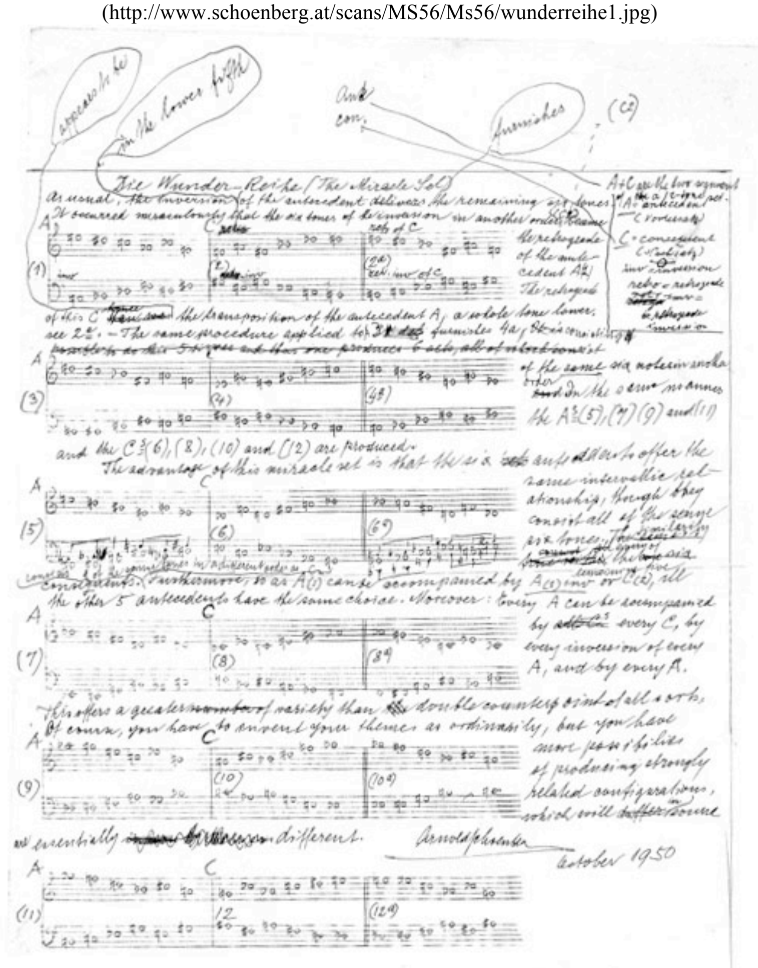
a) Modern Psalm Text with translation by Mark Risinger (2000, 291-92)

Der Erste Psalm	The First Psalm
1) O, du mein Gott: alle Völker preisen dich und versichern dich ihrer Ergebenheit.	1) O, you my God: all people praise you and assure you of their devotion.
2) Was aber kann es dir bedeuten, ob ich das auch tue oder nicht?	2) But what can it signify to you, whether I do likewise or not?
3) Wer bin ich, dass ich glauben soll, mein Gebet sei eine Notwendigkeit?	3) Who am I, that I should believe my prayer to be a necessity?
4) Wenn ich Gott sage, weiss ich, dass ich damit von dem Einzigen, Ewigen, Allmächtigen, All-wissenden und Unvorstellbaren spreche, von dem ichmir ein Bild weder machen kann noch soll. An den ich keinen Anspruch erheben darf oder kann, der mein heissestes Gebet erfüllen oder nicht beachten wird.	4) When I say "God," I speak thereby of the Only, Eternal, Omnipotent Omniscient and Unimaginable, of whom I neither can nor should make myself an image. On whom I may not and cannot make any claim, who my most fervent prayer will either fulfill or disregard.
5) Und trotzdem bete ich, wie alles Lebende betet; trotzdem erbitte ich Gnaden und Wunder; Erfüllungen.	5) And yet I pray, as all living things pray: Yet ask I for grace and miracles; fulfillment.
6) Trotzdem bete ich, denn ich will nicht des beseligenden Gefüls der Einigkeit, der Ver-einigung mit dir, verlustig werden.	6) Yet I pray because I do not want to lose the sublime feeling of unity, of union with you.
7) O du mein Gott, deine Gnade hat uns das Gebet gelassen, als eine Verbindung, eine beseligende Verbindung mit Dir. Als eine Seligkeit, die uns mehr gibt, als jede Erfüllung.	7) O you my God, your grace has granted unto us the prayer, as a bond, a sublime bond with you. As a bliss that gives to us more than any fulfillment.

b) Text Schema for Modern Psalm



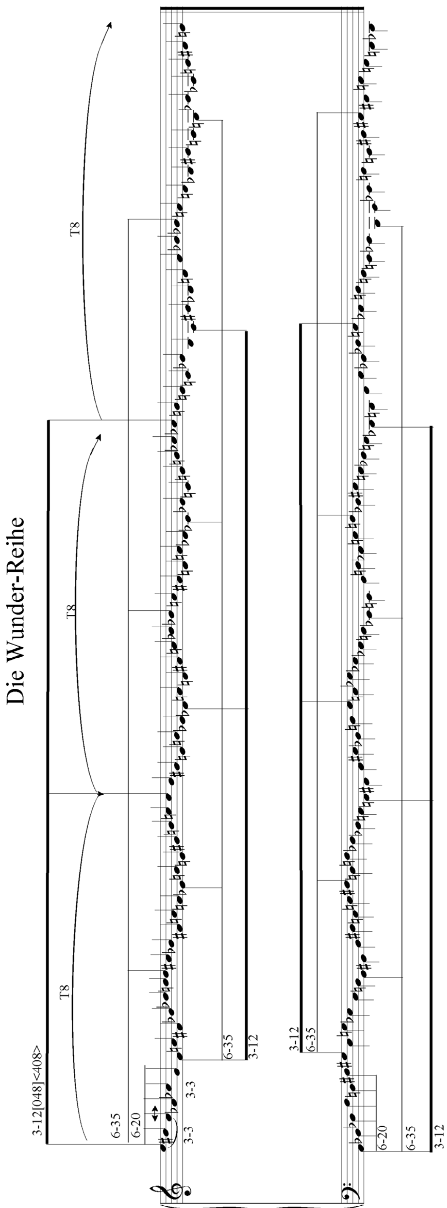
Example 13. Die Wunder-Reihe
(http://www.schoenberg.at/scans/MS56/Ms56/wunderreihe1.jpg)



The fourth paragraph is the central paragraph of the *Psalm*, where the speaker focuses on the unimaginable God and his praiseworthy qualities. Sprecht rightly refers to this fourth paragraph, in which the speaker "praises God [and] the importance of human existence and prayer" as "central not only in location but meaning."[26] It is

[26] Sprecht 1975, 393.

Example 14. Die Wunder-Reihe *redrawn*

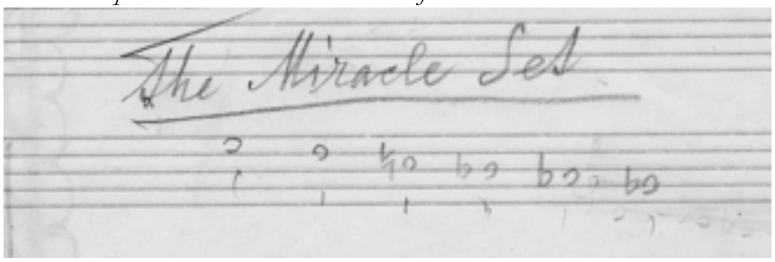


infused with elements of the speaker's reflections from other paragraphs (i.e., God's greatness and the prayer), and is in this way an amalgamation of sections A and B. The three sections of the text distribute sentences symmetrically.[27] This tripartite design and circuitous nature of the text is also reflected in the score.

Example 13 contains a facsimile reproduction of Schoenberg's one-page narrative on *Die Wunder-Reihe* accompanied by an ordered mapping of the hexachord used in *Modern Psalm*. In *Modern Psalm*, the two discrete hexachord halves of the prime row are <4,3,0,8,e,7>(herein H3) and <5,9,6,t,1,2>(herein H1). An uninterrupted realization of Schoenberg's musical chart reveals that the "miracle set" is circuitous. In order to show succinctly the various cycle types embedded within Schoenberg's *Wunder-Reihe,* the succession of twenty-four hexachords has been rewritten in Example 14. The hexachord, which is the primary building block of this circuit, is composed of two inversionally-related 3–3 trichords that continually run through the entire sequence. Every contiguous six-pc segment yields a 6–20 hexachord, and in the upper and lower parts there are twelve consecutive 6–20 hexachords. All the initial notes of transpositionally-related hexachords are beamed together, forming a 6–35 hexachord that can be formed by combining two complementary and canonic invariant hexatonic alley cycles. Schoenberg explicitly draws our attention to the most prominent 6–35 hexachord and "The Miracle Set" by notating the incipit pcs of the prime row regions, as shown in the sketch reproduced in Example 15.

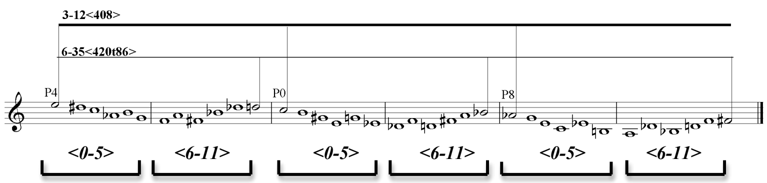
[27] Certain words are emphasized throughout the *Psalm* through repetition, with the most important being God. Schoenberg states "*Gott*" three times in the *Psalm* text; "Gott" – the paramount topic in the work, is equally distributed as the subject header of the first, fourth, and seventh paragraphs, forming a cyclic, tripartite, and symmetrical distribution of the word among the seven paragraphs. This mimics the symmetrical distribution of the iterations of <408> that represent God throughout the work, as the notes of each trichord are likewise symmetrically distributed among the twelve possible pcs.

Example 15. The Miracle Set's transformation 6-35 hexachord



(http://www.schoenberg.at/scans/MS56/MS56/no_no_a.jpg)

Example 16. Possible derivation of incipit of "The Miracle Set"

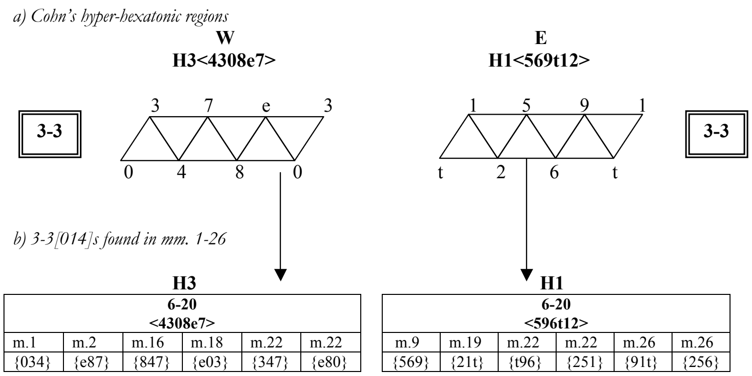


In this sketch Schoenberg shows the descending whole-tone collection accompanied by the title, "The Miracle Set." The derivation of these incipit notes can be understood in two ways. First, in the *Die Wunder-Reihe* sketch, Schoenberg aligns the regions such that each new system is a T_t transposition of the prime row, and the incipit note of each of these T_t related regions forms the whole-tone collection. The second way of perceiving these transformational pcs is shown in Example 16. This time the hexachords of the transformational 6-35 hexachords are drawn exclusively from the T_8-related prime rows P4, P0, and P8. These rows contain all six hexachords of the transformational 6–35 cycle. The three prime hexachords that include H3 (i.e., P4) and its two T_4-related forms (i.e., P0 and P8) occur in order positions *<0–5>*; the three hexachords that include H1 and its two T_4-related forms occur in order positions *<6–11>* in each of the prime rows (although these H3 forms occur in retrograde). The incipit notes of this transformational 6–35 hexachord are beamed together in Example 16, showing that these pcs <4,2,0,t,8,6> also occur as the first and last pcs of the T_8-related regions. This cycle of six prime (and retrograded) hexachords can be understood as expressing the three T_8-related regions. This tripartite structure can be expressed as the union of two staggered T_2-related hexatonic alley cycles and will be herein termed a *transformational 6–35 cycle*.[28] Such cycles prove useful in the analysis of *Modern Psalm* below.

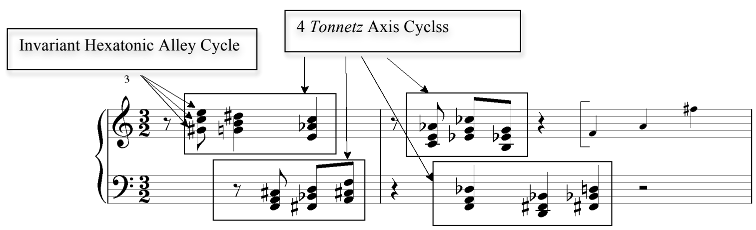
Example 17.a features two 3-3 *Tonnetz* alleys from the modified version of Cohn's hyper-hexatonic system, with the Western region representing H3 and the Eastern region representing H1. Each region contains the six possible spellings of the 3–3 trichord for each 6–20 hexachord. In order to demonstrate that *Tonnetz* axis cycles, *Tonnetz* alley cycles, and invariant hexatonic alley cycles do not occur merely as local events (as seen in Example 10), but also as large-scale events. Example 11.b includes a list of the twelve possible forms of the 3-3 trichord found in the first twenty-six measures of *Modern Psalm* embedded within H3 and H1. Schoenberg uses all twelve possible forms of the 3–3 trichord in measures one to twenty-six, forming two large-scale *Tonnetz* alley cycles, thus traversing all the trichords from both alleys. In measures 28 and 29 he succinctly recapitulates all twelve trichords as shown in the reduction in Example 18. Here, the twelve recapitulated 3–3 trichords occur horizontally (in each of the four rectangles there are three melodic 3-3 trichords), but are aligned in such a way that the vertical sonorities expose all the possible spellings of trichord 3–12, resulting in four pairs of *Tonnetz* axis cycles, and a *Tonnetz* alley cycle (the combination of all the trichords in the bass clef). The upper three voices (written in the treble clef) contain an invariant hexatonic alley cycle. What is remarkable is that the trichordal movement between the hexachords, shown by the neo-Riemannian operators in Example 19, creates a symmetric pattern involving motion in opposing directions. Beginning with the operation LHL in measure 1 of H1

[28] Example 10.a features a pair of alternating T_2-related alley cycles that form a transformational 6–35 cycle. These transformational 6-35 collections are a hallmark of hexatonic-based passages and compositions. See Liszt's *Polonaise I, Die Legende der heiligen Stanislaus*, mm. 98-110 for a "tonal" example that contains a transformational 6-35 cycle (Argentino 2010, 14-15). Allen Forte (1987) discusses Liszt's experimental music, which he claims features tripartite (3-12) divisions of the octave. Forte notes that Liszt's progressive music foreshadows some of the exotic traits in the compositions of the avant-garde composers (Schoenberg, Scriabin, Stravinsky, etc.).

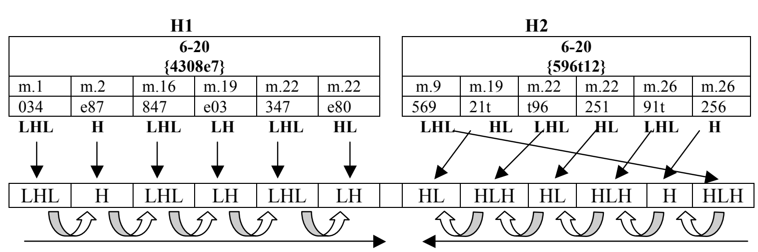
Example 17. Cohn's hyper-hexatonic system and Modern Psalm



Example 18. Reduction of mm. 28-29



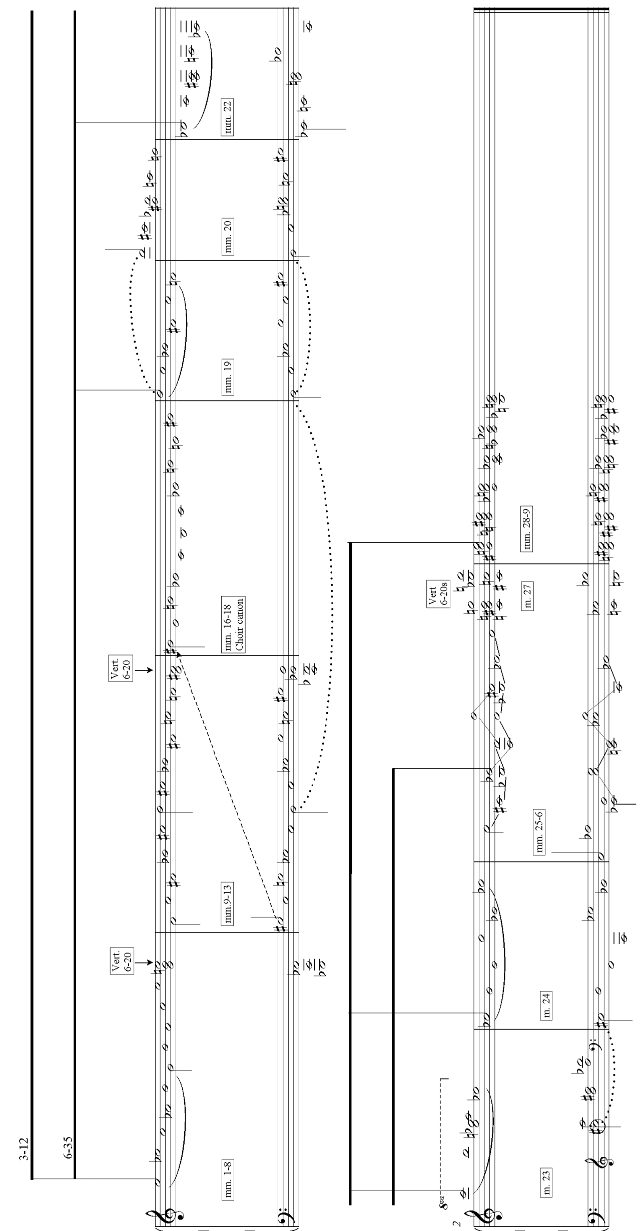
Example 19. 3-3[014]s found in mm. 1-26



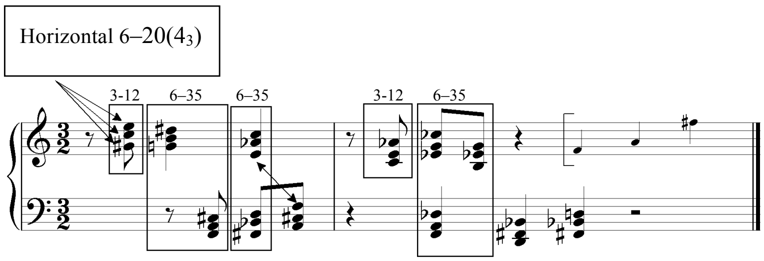
(moving from left to right) and LHL in H2 of measure 9 (moving from right to left), I have redrawn the patterns with arrows below the example, revealing a systematic relationship between both sets of trichords: the neo-Riemannian transformations form an inverse relationship to one another. The symmetry between the trichords and their transformations is also captured in the large-scale cycles of the work.

Example 20 presents a pitch reduction of the first twenty-nine measures of *Modern Psalm*. The first section is divided into three symmetric parts, shown by the beamed 3–12 trichord that shows the incipit pitches of a 6–20(T_4) cycle (i.e., an invariant hexatonic cycle). The six beamed hexachords in this example form a transformational 6–35 cycle that culminates in measure 26, the same measure in which all twelve forms of the 3–3 trichords found within H1 and H2 are depleted. Immediately following the culmination of the 6–35 cycle, the opening hexachord returns at pitch in measures 28–29, accompanied by all twelve 3–3 horizontal trichords arranged as augmented vertical 3–12 trichords, as shown in Example 21. The six hexachords that occur in measures 28–29 recapitulate all six hexachords that occur within the transformational 6–35 cycle. Furthermore, in measures 28–29, the derived 6–35 hexachords and the 3–12 trichord succinctly recapitulate the events of the first 27 measures; these are the only vertical 6–35 hexachords in the work, as shown within the boxed notes in the reduction of measures 28–29 in Example 21. The text that directly precedes measures 28–29 expresses the idea that one should not make an image of God: *Mir ein Bild weder machen kann noch soll*. Prior to the arrival of the verb "should not," which occurs at the end of the sentence, Schoenberg traverses the entire aggregate with vertical representations of H1 and H3, as shown in measure 27 of Example 20. The speaker states "should not" *a capella*, which is followed by a moment of silence. This provides a blatant moment of word painting, with the literal silence in the score reflecting the text: nothingness (i.e., silence) represents Schoenberg's God. The first chord that follows this moment of silence is a vertical 3–12 trichord in the specific form <4,0,8>, and

Example 20. Modern Psalm*, transformational 6-35 cycle, mm. 1-29*



Example 21. Modern Psalm*, mm. 28-29*

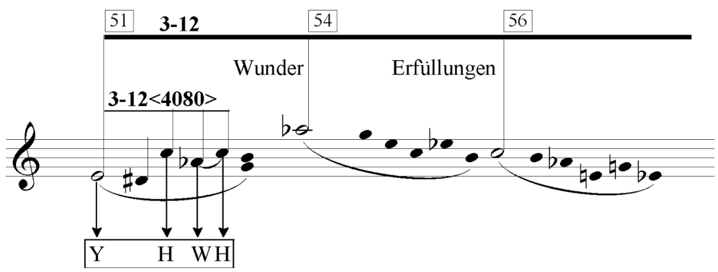


these three pcs are the incipit notes of a vertical 3–12 invariant hexatonic cycle.[29] The three pitches, in the specific form <4,0,8> or <4,0,8,0> represent Schoenberg's unmentionable God. In David Schiller's discussion of *A Survivor from Warsaw*, he notes that trichord <4,0,8,0> represents the Tetragrammaton –the unpronounceable name of God (2003, 103-104). In "Tripartite Structures in *A Survivor from Warsaw*," I demonstrate that the {0,4,8} trichord and tripartite divisions in *A Survivor form Warsaw* symbolically represent God's presence throughout the work. In *Modern Psalm*, the {4,0,8} trichord is once again utilized torepresent God both locally (see the incipit notes of m. 28), and through large-scale devices (the transformational 6-35 cycle that occurs in mm. 1-26).

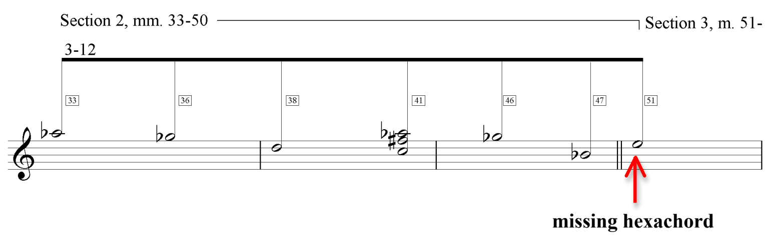
Another example of trichord <4,0,8> representing Schoenberg's ineffable God occurs in the opening measures of the third section of *Modern Psalm*, where H3 is partitioned between clarinet and cello in such a way that the 3–12 trichord (<4,0,8,0>) occurs both ordered and as the highest sounding melody, appearing in the clarinet part as shown in Example 22. While the 3–12 trichord sounds, the speaker states, "And yet I pray, as all living things pray," thus allowing God back into his life.

[29] Immediately following the iteration of H1 in measures 28–29, a restatement of the first appearance of H1 occurs in measures 29–32 accompanied by H3, marking this completion of the aggregate with a *fortissimo* dynamic supporting the text "*erfüllen"* ("fulfilment").

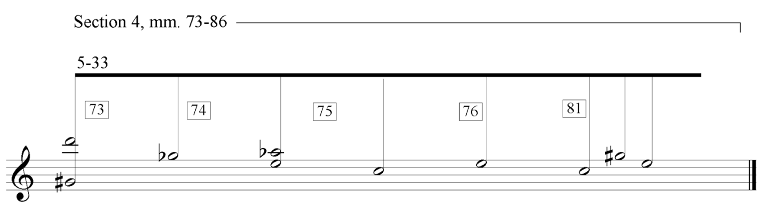
Example 22. 3-12 trichord nesting



Example 23. Modern Psalm, section 2 and prime hexachord reduction



Example 24. Section 4 and prime hexachord reduction



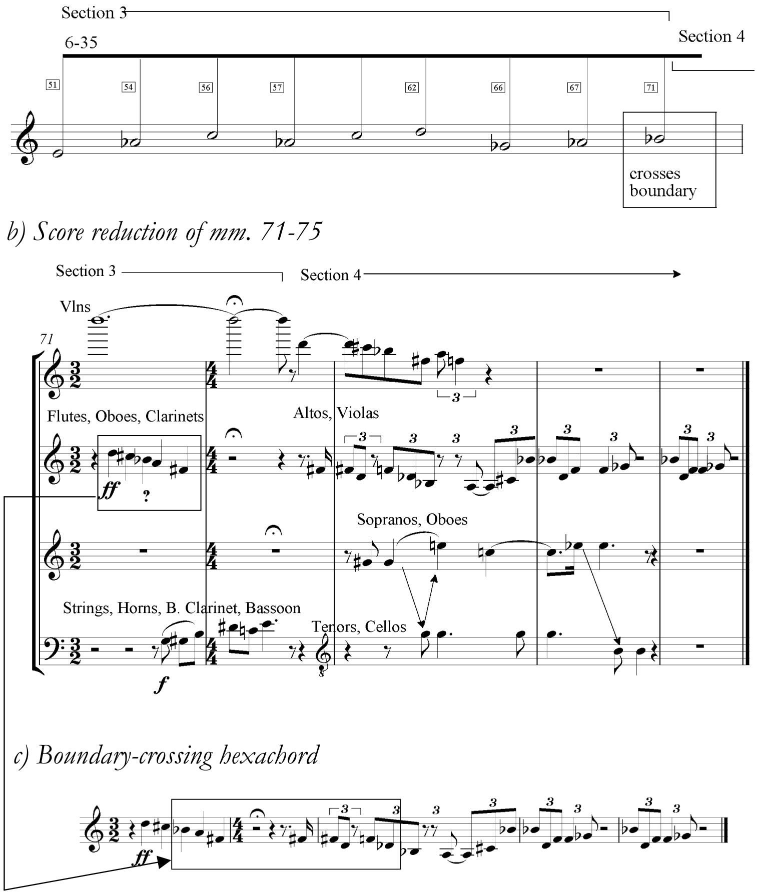
Each note of the initial 3–12 trichord becomes the incipit note of the three related 6–20 hexachords: H3 and its 6–20(T_4) cyclic partners preserve hexachordal content. Shown by the beams in Example 21, the two spellings of the 6–20 hexachord that occur directly following the words "*Wunder*" and "*Erfüllungen*" provide musical fulfillment of the 3–12 trichord motif nesting at the beginning of the section. The miracle hexachord remains a transformational force throughout the remainder of the work.

Although the narrator questions his or her relationship with God, the 6-35 transformational cycle, which contains the <408> trichord that represents Schoenberg's God, is omnipresent throughout *Modern Psalm*. In order to showcase succinctly the remaining transformational 6–35 cycles, the following examples will use the incipit note in place of the entire hexachord. For example, pc 4 will substitute for hexachord <4,3,0,8,e,7>, and pc 2 will substitute for hexachord <2,1,t,6,9,5>. Beginning with section 2 (measures 33–50), five of the six hexachords of the miracle set occur within the movement proper. The missing hexachord, H3, occurs at the very beginning of the third section as shown in Example 23 in m. 51 where the narrator states "*trotzdem bete ich, wie alles Lebende betet*" (Yet I pray, as all living things pray). Note that the moment when the narrator reconciles with God, the missing H3 hexachord appears. Both section 3 (51–72) and section 4 (73–86, incomplete) contain five of the six prime hexachords of *Die Wunder-Reihe*, with both sections missing the prime hexachord <t,9,6,2,5,1>—the hexachord furthest removed from the primary hexachord, both from a cyclic and transpositional (i.e., T_6) standpoint. Example 24 shows how the hexachord never materializes in section 4, thus leaving the transformational 6–35 cycle incomplete (as was the composition). However, the missing hexachord eventually occurs at the boundary of sections three and four through a unique partitioning scheme.

Example 25.a shows the series of hexachords in section 3, and Example 25.b shows a reduction of measures 71-75. The boundary-crossing hexachord is initiated in measure 71. The boxed pentachord sounds *fortissimo* in trumpets, oboes, clarinets, and flutes. Upon first hearing, the first three pitches of this pentachord may initially be mistaken for the hexachord <2,1,t,6,9,5>. However, when the fourth and fifth pitches are sounded it is clear that the ordering of this pentachord does not comply with any segment of pcs found within *Die Wunder-Reihe*. Example 25.c isolates the last three pitches of the boxed pentachord from Example 25.b, and groups them with the four pitches that immediately follow, which sound in alto voices and violas in the fourth section.

Example 25. Modern Psalm*, section 3 and hexachord reduction*

a) Prime hexachords within section 3, mm. 51-72



Retrospectively it becomes clear that this group of six pitches forms the missing prime hexachord <t,9,6,2,5,1>, the hexachord that completes the 6–35 transformational cycle. The conclusion of this cycle recycles the text from the end of section three: "und trotzdem bete ich." In both sections 2 and 3 of the work, the termination of the cycles coincides with the moment at which the narrator decides to "pray nonetheless."

Conclusion

The musical examples in this study feature hallmarks of hexatonic passages, including symmetrical and cyclical divisions of the aggregate and the systematic exhaustion of particular forms of a set-class drawn from a particular pc collection(s). By repurposing the common RPL operators and formulating operation H, I devise a new means of generating cyclic *Tonnetz* spaces by using the asymmetrical trichordal subsets of pitch class set 6–20 as generators of cycles. These cyclic *Tonnetz* spaces, which are based on foreground relationships between set class consistent trichords, are extended in order to capture cyclic relationships between hexachords, especially transformational 6–35 cycles, which Schoenberg uses in order to articulate form and text.

At the heart of the transformational 6–35 cycle, which governs the T_4-related regions of the work, we find pcs <4,0,8>, a trichord that has explicitly been used in *Modern Psalm* and in *A Survivor from Warsaw* to represent Schoenberg's God, who is ineffable within the Jewish tradition. The theoretical constructs that I have derived from my analysis of *Modern Psalm* and "Nacht" may be used as a prism through which to investigate other serial and non-serial works based on a 6–20 hexachord and its subsets.[30] It is my hope that this study provides not only an innovative way of viewing the particular works of this important composer, but also a theoretical framework for investigating other serial and atonal works.

[30] For instance, the focus of such a study could include an investigation of the compositional strategy of systematically depleting forms of a set class through *Tonnetz* cycles.

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