

## Ives's "Piano-Drum" Chords

by

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Among the structures and techniques described by Charles Ives in his *Memos* are various uses of the piano to simulate drum sounds. Ives recalls the origins of this "piano-drum playing" in activities of his youth:

When I was a boy, I played in my father's brass band, usually one of the drums. Except when counting rests, the practising was done on a rubber-top cheese box or on the piano. . . . In practising the drum parts on the piano. . . , I remember getting tired of using the tonic and dominant and subdominant triads. . . . So [I] got to trying out sets of notes to go with or take-off the drums. . . . They had little to do with the harmony of the piece, and were used only as sound-combinations as such.<sup>1</sup>

Ives explains that the chords that serve these purposes were not simply random dissonant combinations, but were specific structures that he had tested and refined, heeding his father's advice to "make

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<sup>1</sup>Charles E. Ives, *Memos*, edited and with appendices by John Kirkpatrick (New York: W. W. Norton, 1972, p. 42). (Kirkpatrick's interpolation.) Recently, Maynard Solomon has expressed doubts about the accuracy of these types of recollections ("Charles Ives: Some Questions of Veracity," *Journal of the American Musicological Society* 40 (Fall 1987):443-70; and "Communication," *Journal of the American Musicological Society* 42 (Spring 1989):209-18). The doubts arise, however, as part of a hypothesis about Ives's idealization of his father that remains to be proven. Before passing judgment, we should require the same "independent confirmation" for Solomon's theories that he expects of conventional hypotheses. Without such confirmation, we are free to accept or reject the basic veracity of Ives's recollections, to presume innocence or guilt, absent of satisfactory proof to the contrary. This study presumes innocence.

some effort to find out what was going on" in his drum imitations (ibid.).

Later, in compositions involving march evocations and march-like rhythms, Ives includes piano chords and rhythms that apparently recall the earlier "Piano-Drum Chords" (PDC). In some cases Ives's memory of a particular work centers on these structures; for example, he traces the history of the *March and Overture, 1776* to the "habit of the piano-drum-playing" (ibid., 83). In other music, the presence of PDCs is highlighted by a performance instruction to play "as a drum," or, simply, in the characteristic sounds and rhythms of the part. Works with passages of this type include those written as "experiments" with specific devices and techniques, as well as music that is, on the whole, more viscerally conceived.<sup>2</sup>

This overt connection between youthful tinkering and mature artistic decisions provides a valuable window for viewing the roots and development of Ives's musical language. By focusing on the PDCs, we can isolate principles of chord construction that reflect some of his earliest inspirations. Because PDCs appear in works of various types, we can gain particularly useful insights into music that may otherwise evade technical characterization. Ultimately, the context of PDCs within all of Ives's pitch structures can suggest avenues toward a broader understanding of his harmonic practices.

The discussion in *Memos* provides ample clues to PDC structure. Ives traces the origins of one type of chord to the imitation of a snare drum with "right-hand notes usually closer together" in opposition to a bass-drum line comprised of "wider chords." He adds, "For explosive notes or heavy accents in either drum, the fist or flat of the hand was sometimes used" (ibid., 42). In another paragraph, he advocates keeping "a different set of notes going in each hand," and warns that "triads and chords without bites

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<sup>2</sup>Kirkpatrick (ibid., 42, note 2) identifies twelve works with passages influenced by PDCs.

were quite out of place, or any combinations that suggested fixed tonalities" (ibid., 43). In more specific terms, he recalls:

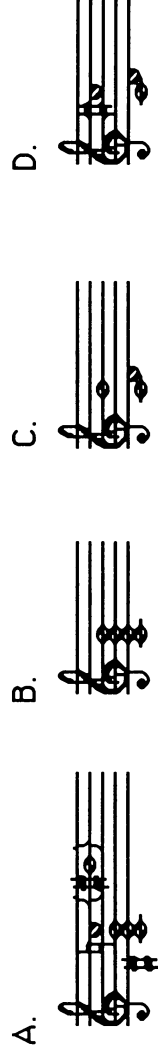
A popular chord in the right hand was Doh<sup>#</sup>-Me-Soh-Doh, sometimes a Ray<sup>#</sup> on top, or Doh-Me-Soh-Ti, and one with two white notes with thumb, having the little finger run into a 7th or octave-and-semitone over the lower thumb note. The left hand would take two black notes on top with thumb, and run down the rest on white or mixed. (ibid.)

These descriptions have been realized in Figure 1. Figure 1A realizes "Doh<sup>#</sup>-Me-Soh-Doh-(Ray<sup>#</sup>)," and Figure 1B realizes "Doh-Me-Soh-Ti." The "two white notes with thumb" underlie Figures 1C and 1D, with the addition of a "7th" and "octave-and-semitone," respectively, above the lowest notes. Figure 1C presumes Ives would favor the major-quality seventh. Ives's less specific description of a left-hand chord is not realized; it might be formed, for example, as D<sup>#4</sup>-C<sup>#4</sup>-A<sup>3</sup>-G<sup>3</sup>-D<sup>3</sup>.

Common to all the structures Ives describes in general and specific terms is an emphasis on pitch-class variety and dissonance cooperating to deny any implications of conventional tonal structures. These features would be most pronounced in a chord made with a "fist or flat of the hand," and would figure prominently in many of the other chords that are generally described. But Ives also proposes more subtle manifestations of these forces, even allowing tertian triads with dissonant "bites" (Figs. 1A and 1B) and chords with fewer notes (Figs. 1C and 1D). Implicit in his descriptions is that sparsely structured but carefully planned PDCs can be as effective as fist/palm chords, given the appropriate context. The sparse chords exhibit no less a concern for pitch-class variety, but simply operate within a more limited range of possibilities and require greater care in their formation.

The pursuit of pitch-class variety in Ives's PDCs embodies one of two approaches. First, a "clustering" approach attempts to saturate a certain area of the pitch-space with as many notes from a certain collection as is possible. A "fist/palm" chord, for example, would apparently be a chromatic cluster, or perhaps an isolation of just white or black keys. Second, a "complementation" approach achieves the pitch-class variety by combining sets that have no pitch-classes in common and that are thus "complimentary" with respect to the larger set that is the sum of their contents. (The larger set may or may not be the aggregate.) Ives describes this formation method in his suggestion to keep a "different set of notes going in each hand." The complementary units are usually differentiated by this type of allocation between hands, and by register--although a complementation between hands within the same register (e.g., white keys vs. black keys) would produce a "cluster" type of chord that is nevertheless divisible into complementary units. Ives also describes a type of complementation within the same hand: chords C and D in Figure 1, for example, are based on a subtler registral separation and dissonant relationship between the two lower thumb notes and the upper notes. Similarly, Ives describes a complementation between white and black keys within the left hand in his suggestion to "take two black keys on top with thumb, and run down the rest white." These principles further apply to the "triadic" type of PDCs in Figures 1A and 1B. Conceptually, the notes of a triad in one of these PDCs (e.g., C<sup>#</sup>-E-G in chord A) can be differentiated from certain non-triad tones (C and (D<sup>#</sup>) in chord A) as a type of "complementation" between the triad tones and the non-triad tones. These PDCs follow special constructional principles to determine the specific group of characteristic non-members and the nature of the differentiation (e.g., the ambiguity in chord A between different triad formations--C<sup>#</sup>-E-G, E-G-C, etc.); they will receive special and separate attention later in this study.

Figure 1. Realizations of PDCs described in *Memos*, p. 43.



PDC clusters generally involve chromatic or white-key saturations, as suggested by a fist/palm chord, or a whole-tone clustering. Figure 2A illustrates complete whole-tone clusters transposed by half steps in the chamber work *Over the Pavements*.<sup>3</sup> Ives specifies the evocation of "short drumbeats," and suggests in the consistent durations a meter that contradicts the bar-lines (beat = dotted quarter-note of the notated meter). The white-key collection clustered in Figure 2B is repeated for a large portion of the chamber work *Calcium Light Night*. Ives again establishes a metric conflict: the standard military cadence rhythm places usual downbeats at weak places in the notated meter, as suggested below the score.

Since many different structures could qualify as PDC "complementations," all chords of this type cannot be as easily linked to specific collections. In the *Fourth of July* (m. 66), for example, is a PDC that reads, from bottom to top, (LH)  $E^b$ -F-B-D-E, (RH) F-G-C- $D^b$ -F-G. Typically, the pitch-class non-intersection is not exact, since pitch-class F is common to both hands; also, F and G appear twice within the right hand. Still, the chord easily achieves the desired effects of dissonance and pitch-class variety, altogether stating pitch-classes {B,C, $D^b$ ,D, $E^b$ ,E,F,G}.

In two special cases, complementations recall the common cluster types, juxtaposing opposite whole-tone collections or groupings of white and black keys. Because of the relationship between this sub-group of PDC complementations and the clusters,

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<sup>3</sup>In this passage from *Over the Pavements* there is one discrepancy between the pattern of the chord structures and the printed score: the final left-hand trichord in the published score is  $D^2$ - $E^2$ - $F^2$ , thus disrupting the pattern of pure whole-tone collections. The origins of this inconsistency can be traced to the sketch, where Ives uses a short-hand in m. 48, indicating a continuation of whole-tone clusters as in the previous measures by drawing a vertical line from the right-hand trichord to what would be the bottom note of the left-hand trichord. For the last chord he writes  $D^2$  as the bottom note. Presumably, he would have discontinued the short-hand if he had wanted a different collection here, and he intended to write  $E^{b2}$ , which would be consistent with the pattern. Figure 2A notates the final trichord consistently with the preceding chords, as Ives had apparently intended.

**Figure 2. PDC Clusters.**

A. *Over the Pavements*, piano mm. 46-48.

Figure 2A shows a musical score for piano, measures 46-48. The score is written for two staves (treble and bass clef). Measure 46 is marked with a triangle symbol (Δ) above the staff. Measure 47 is marked with a triangle symbol (Δ) above the staff. Measure 48 is marked with a triangle symbol (Δ) above the staff. The score includes a bracketed section labeled "(as short drumbeats, no pedal)" spanning measures 46-48. The notation includes various notes, rests, and dynamic markings.

B. *Calcium Light Night*, piano II mm. 10-11.

Figure 2B shows a musical score for piano II, measures 10-11. The score is written for two staves (treble and bass clef). Measure 10 is marked with a triangle symbol (Δ) above the staff. Measure 11 is marked with a triangle symbol (Δ) above the staff. The score includes a bracketed section labeled "left, right, left" spanning measures 10-11. The notation includes various notes, rests, and dynamic markings.

we can more specifically describe the structure and possibilities for this type. First, we represent each cluster type as a pitch-class collection: whole-tone (WT) even (pcs 0,2,4,6,8,10) or odd (pcs 1,3,5,7,9,11), and white-key (WK) (pcs 0,2,4,5,7,9,11). Each collection is the "parent" of a "family" of pitch-class sets that can be formed from its members. For example, the even WT family includes any combination of even-numbered pitch-classes--any member of Forte set-classes 2-6, 3-6, 3-8, 3-12, 4-21, 4-24, 4-25, 5-33, or 6-35 that is realized with even-numbered pitch-classes.<sup>4</sup> The odd WT family includes any combination of odd-numbered pitch-classes, and the WK family includes any combination of white-keys--white-key realizations of subsets of Forte-class 7-35. Excluded from each family is a "complement family" of pc sets that can be formed from non-members; the two WT families are complement families of each other, and the WK family excludes a family of black-key (BK) combinations as its complement family. Thus PDC complementation juxtaposes a member of a family against a member of its complement family--WT odd against WT even, or BK against WK. Literally, the "complementation" occurs within the all-inclusive set that is the sum of its contents of the two opposing family members. Conceptually, however, the complementation remains behind the scenes between the two parent collections, which are complementary with respect to the aggregate.

Musical applications often establish prominence for a certain collection, suggesting an opposition of this "prime collection" against a member of its complement family. (Of course, the opposing collections may also occur on equal terms.) In the "roll-offs" from the "Hawthorne" movement of the "*Concord*" *Sonata* shown in Figure 3A, a grace-note figure precedes and embellishes six simultaneities that are members of the odd WT family, establishing these as the "prime collections" for the passage. The

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<sup>4</sup> Allen Forte, *The Structure of Atonal Music* (New Haven and London: Yale University Press, 1973).



3. PDC Complementations.

A. *Piano Sonata No. 2*, "Hawthorne," bottom p. 36 (AMP edition)

fff

[as the drum-corps]

B. *Fourth Symphony*, II, Primo orchestra piano mm. 123-124.

123 124

A

(as a Drum)

first two simultaneities (the chords at the ends of the phrase markings) are (G,C<sup>#</sup>,G,B,E<sup>b</sup>,F), a five-pitch-class member of the odd WT family (pc 11,1,3,5,7; SC 5-33); the last four simultaneities (accented with inverted "roof-tops") omit the lower G<sub>3</sub> and C<sup>#</sup><sub>4</sub> from this set, leaving a four-pitch-class family member (SC 4-24). (The upper G<sup>#</sup> is attacked before the simultaneity, as part of the grace-note figure.) Instead of the simultaneous combination of family members that is usual within PDCs, the grace-notes provide the complement-family opposition, including pitch-classes F<sup>#</sup>, G<sup>#</sup>, and A<sup>#</sup>. (The grace-note figures preceding the fourth and fifth chords temporarily include a G from the prime collection.)

Though some WK/BK complementations value higher numbers of pitch-classes, as if juxtaposing a section of black-keys above the WK cluster in Figure 2B, the excerpt from one of the piano parts in the *Fourth Symphony* shown in Figure 3B illustrates a subtler result of this approach. In supporting the rhythm of the snare drum line, this passage distributes the WK family member (A,B,C,E,F) within the right hand and the BK family member (A<sup>#</sup>,C<sup>#</sup>,F<sup>#</sup>) within the left hand. The triad formations, F<sup>#</sup>M in the left hand and FM plus E and B in the right hand, are distorted by close chromatic associations--all triad tones are neighbored by half-steps, and the complete pitch-class content of the passage is (E,F,F<sup>#</sup>/A,A<sup>#</sup>,B,C,C<sup>#</sup>), placing every note within a three- or five-member chromatic set.

What is true of the relationship between FM and F<sup>#</sup>M in Figure 3B is also true of the "triadic" PDCs realized in Figure 1A and 1B: triad tones are related by interval-class 1 to other tones. In Figure 1A a C<sup>#0</sup> triad is accompanied by a note (C) that is interval 11 higher than the root, and possibly by an additional note (D<sup>#</sup>) that is interval 11 higher than the third. Or, the same chord could be explained as a CM triad accompanied by a note (C<sup>#</sup>) that is interval 11 lower than the root, and possibly by an additional note (D<sup>#</sup>) that is interval 11 higher than the third. Similarly, Figure 1B could be

viewed as CM plus B (interval 11 from root) or Em plus C (interval 11 from fifth). For convenience, these relationships can be termed "half-steps" between pitch-classes, so that the C and D<sup>#</sup> in the first chord can be described as "half-steps below" the root and third, respectively, of C<sup>#0</sup>, and B as a "half-step below" the root of CM (or C<sup>#</sup> in chord A is a half-step above the root of CM, etc.). Implied in Ives's comments and musical usages is that any triad can form the basis for this unique type of PDC, if at least one added half-step adds the "bite" that an effective PDC must have (*Memos*, 43, quoted above). The bite provides an "embellishment" of a triad tone, producing a "chromatically embellished triad" (CET).

Ives's frequent use of CETs in his piano-drum writing, and his explorations of numerous possibilities for this type of chord structure, invite closer study of this sub-category of PDC. Using previous concepts and terminology, we can establish a certain triad within the chord as the "prime collection." In many cases, there will be an ambiguity of different triad formations within the same chord, as there is between C<sup>#0</sup> and CM in Figure 1A. Given such choices, the designation of a prime collection may be influenced by context, or may be made arbitrarily; ultimately, these ambiguities will dissolve in a generalization of CET equivalence classes. The complement family for a prime-collection triad cannot simply include any non-triad tone, since not all of these will qualify as "chromatic embellishments"--in the chord of Figure 1A, for example, a B<sup>b</sup> would not provide a half-step embellishment to any triad tone. Instead, the complement family is limited to the three upper and three lower chromatic embellishments, combining with the three triad tones to amass a nine-tone aggregation of pitch-class possibilities. The embellishments form same-quality triads half-steps above and below the prime collection--suggested in Figure 3B, for example, is an embellishment of FM (RH) with F<sup>#</sup>M (LH) and the root and fifth of EM (RH m. 124 beat 2). Though the number of

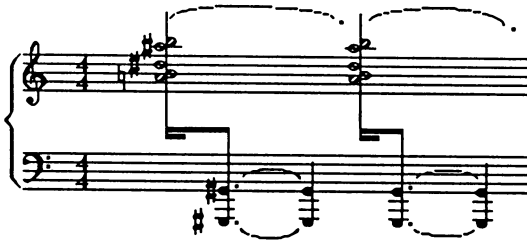
embellishments will be variable, a chord must contain a complete triad as prime collection in order to be a "CET."

For purposes of further examining the triadic foundations of CETs, the following notation will describe any CET. The complete nine-tone array will be represented by the symbol "135/135," where the numerals represent chromatic embellishments of corresponding triad tones--numerals preceding the virgule specify upward embellishments of chord tones "1" (= root), "3" (= third), and "5" (= fifth), and numerals following the virgule specify downward embellishments of those chord tones. In effect, the virgule represents the prime-collection triad, which is always present. An "embellishing pattern," the list of embellishments for a given CET, will select from this roster of available embellishments, adding the numeral 0, for clarity, when no embellishments are added in a given direction, and preceding the complete symbol with an indication of the root and quality of the prime collection. Thus Figure 1A is C<sup>#</sup>0 plus a downward embellishment of the root and no upward embellishments: C<sup>#</sup>:0/1 (or CM plus an upward embellishment of the root, CM:1/0). Figure 1B is CM plus a downward embellishment of the root: CM:0/1 (or Em:5/0). The content of Figure 3B is FM:135/15.

Ives usually places CETs in the moderate-to-low register of the piano, low enough to obscure any strong tonal implications, but high enough to preserve a triadic presence. Chromatic embellishments appear directly adjacent to chord tones or separated in register. Figure 4A gives a typical example--although somewhat high in pitch--taken from the early measures of the song *General William Booth Enters Heaven*. Reflecting the first line of text, "Booth led boldly with his big bass drum," the song opens and closes with PDCs staggered rhythmically between hands as shown in Figure 4A. The chord includes the G<sup>#</sup>-minor triad as the prime collection, plus A, an upward embellishment of the root, and A<sup>#</sup>, a downward embellishment of the third: G<sup>#</sup>m:1/3.

**Figure 4. Selected CETs.**

**A. *General William Booth Enters Into Heaven*, piano m. 11.**



G<sup>#</sup>-B-D<sup>#</sup> plus A plus A<sup>#</sup> = G<sup>#</sup>m: 1/3

**B. *Calcium Light Night*, piano 1 mm. 1-2.**



RH: C<sup>#</sup>-E-G plus C plus F<sup>#</sup> = C<sup>#</sup>°: 0/15

LH: G<sup>#</sup>-B-D<sup>#</sup> plus A<sup>#</sup> = G<sup>#</sup>m: 0/3

Among Ives's variable treatments of CETs are combinations of different chords that present greater pitch-class variety, and ultimately obscure triadic presences. Figure 4B gives the opening of the chamber work *Calcium Light Night*, exhibiting another application of the military cadence rhythm (cf. Fig. 2B; also, see the beginning of *General William Booth . . .*). The first chord, repeated as the second, fourth, and fifth chords in the two-bar pattern, is comprised of  $C^{\#0}:0/15$  in the right hand, a variant of the chord described in *Memos* and realized in Figure 1A, and  $G^{\#m}:0/3$  in the left hand. The hands have no pitch-classes in common, and the complete effect is one of pitch-class saturation rather than triadic embellishment. Thus it is a "complementation" between the hands, combining CETs that are themselves reliant on a "complementation" within each hand between the triad and its embellishments. The same effect often results from juxtaposition of a CET with any group of different pitch-classes; in Ives's *Piano Trio*, for example, CETs are set against members of whole-tone families.<sup>5</sup>

With the multiplication of variants and alternatives, the tones of the triad in these chords retreat into a general context of pitch-class saturation, obscuring potentially meaningful distinctions between the structure of a CET as opposed to the non-triadic PDC types. To preserve a sense of "chromatic embellishment," the need arises for a refined definition of a CET that recognizes the importance of the triadic presence. Indeed, the nine-pitch-class summary of the prime collection and the available upper and lower embellishments itself permits an inappropriately large number of non-triad tones that could too easily obscure the prime collection. It also lacks appropriate limitations, since many different structures, whether or not clearly formed around a triad, could be associated with, and included in, a pitch-class summary of this size. These

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<sup>5</sup>This occurs in mm. 47-64 of the 2nd mvt. The left-hand reiterates  $Am:0/1$  "as a bass drum" while the right-hand answers with afterbeats from one or the other WT family.

issues arise with the single different simultaneity in Figure 4B--the third chord in the pattern, at the beginning of measure 2. Theoretically, we can identify all the notes as embellishments of the  $G^\#$ -minor triad: the left-hand states  $G^\#m:0/3$ , as in the other chords in the pattern, while the right-hand adds upper and lower embellishments to the root and an upper embellishment to the fifth, totalling  $G^\#m:15/13$  for the entire chord. But like the other chords in the pattern, the effect is more pitch-class saturation than triadic embellishment.

Generally, these imperfections arise when 1) embellishments outnumber chord tones and 2) a single chord tone is embellished in both directions, a "dual embellishment." Both conditions exist in the  $G^\#m:15/13$  of Figure 4B, since there are four embellishments, and the root  $G^\#$  is dually embellished by A and G. (Dual embellishments are inevitable in seven-tone sets and larger.) We can preserve the integrity of a CET by insisting that there may be a maximum of three embellishments, and that no chord tone may be embellished more than once. That is, we limit the size of a true "CET" to six pitch-classes, and stipulate that, in the symbol for the embellishing pattern, the same numeral cannot appear on both sides of the virgule. PDCs that fall outside these strictures will more appropriately fall into other constructional categories.

Certainly, this redefinition will reduce the possible CETs to a corpus of meaningful size, but will also allow a gray area between categories. For example, the restrictions dictate that the  $G^\#m:15/13$  in Figure 3B could be converted to a "CET" by removing the A (leaving  $5/13$ ) or the G ( $15/3$ ). This adjustment would probably not alter the chord's effect to an appropriate extent. On the other hand, the registration and voicing of the chord contribute to this result, and one can easily envision another presentation of  $G^\#m:5/13$  or  $15/3$  that would highlight the triadic presence. Also, potentially confounding are structures with only a dual embellishment and no other embellishments ( $1/1$ ,  $3/3$ , or  $5/5$ ).

Because such a chord has more chord tones than embellishments, it might retain its triadic character in the appropriate musical setting. In most cases, however, structures of this type obscure the triadic basis and are best allied with the more chromatic PDCs.<sup>6</sup>

Recognizing the new restrictions, we can now summarize all possible CETs. The possible cardinalities, which are self-evident in the embellishing patterns, are four (one embellishment), five (two embellishments), and six (three embellishments). For each cardinality, the possible embellishing patterns are:

tetrads: 0/1 0/3 0/5 1/0 3/0 5/0

pentads: 0/13 0/15 0/35 1/3 1/5 3/1 3/5 5/1 5/3 13/0 15/0  
35/0

hexads: 0/135 1/35 3/15 5/13 13/5 15/3 35/1 135/0

Within each cardinality, the list is ordered ascending numerically, as if the virgule were a decimal point.

This summary offers several points of useful information about CETs. First, we can infer the patterns that are excluded by the restriction against dual embellishment. While no tetrads are excluded, 1/1, 3/3, and 5/5 are excluded from pentads, and the hexad list has been reduced substantially, leaving only eight possible patterns. The list also offers a convenient notation of inclusion relations, assuming that the prime collection is the same, in the recurrence of the same embellishment(s) in a larger chord: 3/0 would be a subset of 3/1, which would be a subset of 35/1, etc. Still, equivalent embellishing patterns do not necessarily reflect other types of equivalences or inclusions, since different qualities of prime collections will produce different pitch-class sets from the same

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<sup>6</sup>A notable exception is a PDC passage in *Putnam's Camp* (mm. 53-57), including Cm:3/3 and E<sup>b</sup>M:1/1 in which triads are not badly obscured, largely due to sparse, clear voicings.



embellishing patterns. And, as we have seen, different embellishing patterns may produce identical pitch-class sets. To resolve these ambiguities, we can recognize the  $T_n/T_nI$  equivalence classes that account for each pattern applied to any quality triad. We can, in effect, establish a CET set-class vocabulary.

The numbers of equivalent patterns and triad qualities within a class are variable. As shown above, Figure 1A (without the  $D^\#$ ) can be interpreted as  $C^\#0:0/1$  or  $CM:1/0$ ; more generally, any  $0/1$  embellishment of a diminished triad, or "d:0/1," will be equivalent to any  $M:1/0$ . But, for example, d:0/5 (e.g.,  $\{C^\#, E, G, F^\#\}$ ) could not be explained as an embellishment of any other quality triad. Also, the  $T_n/T_nI$  equivalence classes encompass inversional equivalences that will be reflected in the embellishing patterns. For example, d:0/5 (e.g.,  $\{C^\#, E, G, F^\#\}$ ) is inversionally equivalent to d:1/0 (e.g.,  $\{C^\#, D, E, G\}$ ) within set-class [0136]. And the equivalence between major and minor triads (SC [037]) will be similarly displayed: M:0/5 (e.g.,  $\{C, E, G, F^\#\}$ ) is equivalent to m:1/0 (e.g.,  $\{C, E^b, G, C^\#\}$ ) within SC [0137]. Every embellishing pattern has an inverse that is  $T_nI$ -equivalent, related as follows: 1) A numeral on one side of the virgule inverts to its mod-6 inverse on the other side ( $1 \leftrightarrow 5$ ,  $3 \leftrightarrow 3$ ,  $0 \leftrightarrow 0$ ); and 2) the quality inverts according to expected set-class equivalences--diminished and augmented do not change (since they are equivalent to themselves under inversion), and major and minor invert to each other. In the CETs above, d:0/5 inverts to d:1/0, because 0 stays 0 and moves to the right, 5 becomes 1 and moves to the left, and the quality stays diminished. M:0/5 above inverts to m:1/0 with the same movement of embellishments ( $0 \rightarrow 0$ ,  $5 \rightarrow 1$ ), but adding a change in quality from major to minor. And  $m:3/1 \leftrightarrow M:5/3$ ,  $M:1/5 \leftrightarrow m:1/5$ ,  $A:15/3 \leftrightarrow A:3/15$ ,  $d:135/0 \leftrightarrow d:0/135$ , etc.

Figure 5 lists the  $T_n/T_nI$  equivalence classes for all embellishments of any quality triad. The embellishing patterns are aligned vertically according to the quality of the prime collection,

**Figure 5.** CET  $T_n/T_nI$  Equivalence Classes

prime form	Forte Set-class	embellishing pattern			A:
		d:	M:	m:	

1. Embellishments of single triads.

A. Prime collection [036] (Forte-class 3-10).

[0136]	(4-13)	<0/5,1/0>
[0236]	(4-12)	<0/3,3/0>
[01236]	(5-4)	<1/3,3/5>
[01346]	(5-10)	<0/35,13/0>
[01356]	(5-12)	<1/5>
[012356]	(6-3)	<1/35,13/5>

B. Prime collection [037] (3-11).

[0137]	(4-29)	<0/5, 1/0>
[0158]	(4-20)	<0/1, 5/0>
[0237]	(4-14)	<3/0, 0/3>
[0347]	(4-17)	<0/3, 3/0>
[01237]	(5-5)	<3/5, 1/3>
[01568]	(5-20)	<0/15, 15/0>
		<3/1, 5/3>
[012378]	(6-38)	<3/15, 15/3>

2. Embellishments of two triads.

A. Prime collection [036] (3-10) or [037] (3-11).

[0147]	(4-18)	<0/1,5/0>	<1/0, 0/5>
[01347]	(5-16)	<0/13,35/0>	<1/3, 3/5>
			<0/35, 13/0>
[01367]	(5-19)	<0/15,15/0>	<1/5, 1/5>
[01457]	(5-18)	<3/1,5/3>	<13/0, 0/35>
[012367]	(6-5)	<3/15,15/3>	<13/5, 1/35>
[013467]	(6-13)	<0/135,135/0>	<1/35, 13/5>

**Figure 5 (continued):** CET  $T_n/T_nI$  Equivalence Classes

prime form	Forte Set-class	embellishing pattern			
		d:	M:	m:	A:
B. Prime collection [037] (3-11) or [048] (3-12).					
[0148]	(4-19)		<5/0, 0/1>	<1/0,0/5> <5/0,0/1> <3/0,0/3>	
[01348]	(5-17)		<35/0, 0/13>	<1/3,3/5> <5/1>	
[01458]	(5-21)		<5/3, 3/1> <1/5, 5/1> <0/13, 35/0>	<0/13,35/0> <0/15,15/0> <0/35,13/0>	
[014589]	(6-20)		<5/13, 35/1>	<0/135,135/0>	

3. Embellishment of any triad.

[01478]	(5-22)	<5/1>	<15/0, 0/15>	<1/5> <3/1,5/3>	
[013478]	(6-19)	<5/13,35/1>	<135/0,0/135> <15/3, 3/15> <35/1, 5/13> <0/135,135/0>	<1/35,13/5> <3/15,15/3> <5/13,35/1>	

shown in the four columns in the right-hand half of the page. Angle brackets surround patterns that are inverse-equivalent. Prime forms, Forte's set-class labels, and the corresponding embellishing patterns are organized into categories according to the number of different qualities that are possible as the prime collection: category 1 lists set-classes with one quality as the prime collection, category 2 lists set-classes with two possible prime collection qualities, and category 3 lists the two set-classes with any possible triad quality as the prime collection. Categories 1 and 2 have sub-groupings showing which quality serves as the prime collection.

Conceptually, the categories arise from testing a prime collection and its allowable embellishments--with the restrictions on size, the possibilities for embellishment grow smaller as more embellishments are added. When more than one quality can be a prime collection, the restrictions interact between qualities, since an allowable embellishment for one quality may be disallowed for another. The chart does not necessarily illustrate set-class inclusions within categories, although it does, of course, list common triadic subsets. Within category 1B, for example, the tetrads [0137] and [0237] are both included in both pentads of this category, but [0158] is included in only one of the pentads (in [01568]), and [0347] is included in neither of the pentads. [0347] is, however, included in [01347] of category 2A and in [01458] of category 2B.

Ives's usages favor particular prime collection qualities and embellishing patterns, thus favoring certain categories and set-classes. SC 6-19 (category 3) is common, frequently formed from a juxtaposition of two major or two minor triads (0/135 or 135/0). For example, Figure 3B includes an initial juxtaposition of FM and F<sup>#</sup>M to form this hexad type. In general, Ives favors lower embellishments of thirds and fifths, and upper embellishments of roots. In Figure 4A, G<sup>#</sup>m:1/3 illustrates SC 5-5, a pentad from category 1B and thus with no ambiguity as to the prime collection. The juxtaposed CETs in Figure 4B include SC 4-14 in the left hand,

also from 1B, and SC 5-19 in the right hand, a 2A pentad that could be read as CM:1/5 or C<sup>#0</sup>:0/15.

As an example that will illustrate other tendencies in Ives's set-class selection, and encapsulate issues from all of his PDC writing, Figure 6 gives an extended PDC passage from the *Fourth of July*. These measures are part of an intensification of quoted patriotic tunes and opposing tonalities culminating in an "explosion" of musical fireworks at the work's conclusion. The musical ideas include typical PDC constructions and variations on PDC principles. The excerpt begins with a triadic interplay at the beginning of measure 93 that concludes the previous passage, followed by three main sections: 1) the PDC repetitions beginning on beat 4 of m. 93 and extending to the middle of m. 95, followed by a short recall of triads (immediately surrounding the m. 96 bar-line); 2) mm. 96-98, PDC-like structures in the left hand and chromatic syncopations in the right hand; and 3) mm. 99-113, an extended use of PDCs in the left hand to accompany a variant of "Yankee Doodle" in the right hand. The quoted tune begins in B<sup>b</sup>M and without substantial variation through the first half of m. 102. Thereafter, the tune reappears in varied form in CM (mm. 102-107), and then with additional variations and fragmentations in FM (mm. 107-112).

The triadic interplay in m. 93 and surrounding the m. 96 bar-line generally associates roots by half steps, thereby recalling the category-3 0/135 or 135/0 embellishing patterns and SC 6-19. The intervening PDC passage (m. 93 beat 4 through the end of m. 95) begins with the notes (E,F<sup>#</sup>,G<sup>#</sup>,A<sup>#</sup>,D), in the left hand, a "cluster" PDC from the even WT family (SC 5-33). The next chord (last chord of m. 93, LH), retains (F<sup>#</sup>,G<sup>#</sup>,A<sup>#</sup>,D) from the previous chord but substitutes D<sup>#</sup> for E, thus introducing an odd pitch-class, i.e. a non-family-member. As PDCs, the left-hand continues this alternation between the 5-33 and the chord with the single non-member. The right hand offers a linear presentation of 5-33 from the odd WT family--in order of presentation, (F,G,A,C<sup>#</sup>,D<sup>#</sup>)--but

Figure 6. *The Fourth of July*, piano mm. 93-113

The musical score for 'The Fourth of July' by John Williams, piano, measures 93-113. The score is written for piano and features complex rhythmic patterns, including triplets and sixteenth notes. The key signature is one flat (B-flat major/D minor). The score is divided into two systems, with measures 93-98 in the first system and measures 99-113 in the second system. The notation includes various musical symbols such as notes, rests, and dynamic markings.

Measures 93-98: The first system contains measures 93 through 98. It begins with a triplet of eighth notes in measure 93, followed by a series of sixteenth notes and eighth notes. Measure 94 features a triplet of eighth notes. Measure 95 has a triplet of eighth notes. Measure 96 has a triplet of eighth notes. Measure 97 has a triplet of eighth notes. Measure 98 has a triplet of eighth notes.

Measures 99-113: The second system contains measures 99 through 113. It begins with a triplet of eighth notes in measure 99, followed by a series of sixteenth notes and eighth notes. Measure 100 features a triplet of eighth notes. Measure 101 has a triplet of eighth notes. Measure 102 has a triplet of eighth notes. Measure 103 has a triplet of eighth notes. Measure 104 has a triplet of eighth notes. Measure 105 has a triplet of eighth notes. Measure 106 has a triplet of eighth notes. Measure 107 has a triplet of eighth notes. Measure 108 has a triplet of eighth notes. Measure 109 has a triplet of eighth notes. Measure 110 has a triplet of eighth notes. Measure 111 has a triplet of eighth notes. Measure 112 has a triplet of eighth notes. Measure 113 has a triplet of eighth notes.

Dynamic markings: The score includes dynamic markings such as  $AM:0/5$ ,  $FM:0/5$ , and  $EM:0/5$ , which are likely indicating the dynamic range or volume of the piano.

(Figure 6 cont.)

This musical score consists of two systems of staves, each with a grand staff (treble and bass clefs) and a single staff below. The first system covers measures 99 to 101. Measure 99 is marked with a brace and a 'p' dynamic. Measure 100 features a 'C<sup>o</sup>:1/5' annotation. Measure 101 is marked with a 'T5(P)' annotation. The second system covers measures 102 to 104. Measure 102 is marked with a brace and a 'BbM:0/15' annotation. Measure 103 features a 'T7(P)' and 'F#M:1/5' annotation. Measure 104 is marked with a 'T7(P)' and 'FM:3/5' annotation. The notation includes various musical symbols such as notes, rests, and dynamic markings.

99 100 101

$C^o:1/5$   $T5(P)$

102 103 104

$BbM:0/15$   $T7(P)$   $F\#M:1/5$   $FM:3/5$

(Figure 6 cont.)

105 106 107 7 8

108 109 110 5

TO(P) FM:1/5 FM:1/1 FM:1/15

BbM:1/5 plus pc G P BbM:1/0 plus pc C P BbM:0/1 FM:1/15



(Figure 6 cont.)

111

3

112

113

4

2

P plus pc G

Ebm:15/5

The image shows a musical score for a piano and percussion ensemble. The score is written on two staves. The top staff is for the piano, and the bottom staff is for the percussion. The piano part features a complex melodic line with many accidentals and a large bracketed section. The percussion part includes a series of rhythmic patterns, some of which are marked with 'P' and 'pc G'. The score is labeled with measures 111, 112, and 113. The key signature is E-flat major (Ebm) and the time signature is 15/5. The tempo is marked as 'P plus pc G'.

adds the non-family-member  $B^b$  (third note of m. 94, RH). Later, the right hand adds other non-members C and D (m. 95 beat 2), but these are in the register of omitted left-hand notes, and seem to separate themselves from the preceding right-hand material. Thus the passage juxtaposes WT families plus single non-members: the right-hand pitch-class content is  $\{1,3,5,7,9\}$  plus  $\{10\}$  and the left hand is  $\{2,4,6,8,10\}$  plus  $\{3\}$ . The hands unfold a complementation of the type often found within a single PDC.

The next portion discontinues "piano-drum" imitations, but maintains the spirit of PDC constructions. Certainly, this begins with the triadic interplay, and then continues in the five chords beamed together in the left hand (m. 96). The first of these is  $(C^\#, E, A, D^\#)$ , or AM:0/5, a category 1B tetrad (SC 4-29). This is transposed by  $T_9$  to derive the third chord  $(F^\# M:0/5)$ , beginning a sequence that ends with another  $T_9$  transposition to the fifth chord  $(E^b M:0/5)$ . The second and fourth chords, also part of the sequence, suggest subsets of CETs; for example,  $(B, D^\#, A^\#)$  (middle of m. 96, LH) could be BM:0/1 with the fifth of the prime-collection triad omitted. While this would not qualify as a CET, it could easily be perceived as a subset, as part of a CET "family" analogous to a WT or WK family. In mm. 97-98, the left hand continues with such subsets, serving to emphasize the greater dissonances by omitting chord tones. For example, the left-hand trichord  $(D^\#, G^\#, E)$  in m. 97, which begins a  $T_{10}$  sequence to the next two left-hand trichords, could be explained as the root, third, and /1 of EM, or the root, fifth, and 5/ of  $G^\# m$ , etc. Throughout this passage, the right hand similarly enhances dissonance in a general chromatic descent (with octave displacements and occasional detours) that frequently adds additional embellishments to the lower CETs or CET subsets.

The accompaniment to "Yankee Doodle" in the final section (mm. 99-113) comprises one of Ives's longest PDC passages. Echoing the first key of the tune, the left hand begins with the

category 2A pentad SC 5-19, best viewed as  $B^bM:1/5$  to correspond with the key of the melody at this point (the other choice is  $B^0:0/15$ ). This CET is labelled "P" in Figure 6. From here to m. 105, P or  $T_n(P)$  is reiterated as a kind of reference sonority, with other CETs interspersed, as illustrated below the score in Figure 6. (Except for mm. 112 and 113, each label applies until a new one appears.) Because the two most prominent transpositions of P are  $T_5$  (m. 101) and  $T_7$  (m. 102), the entire passage mirrors a tonal framework, beginning with P as "tonic" ( $B^bM:1/5$ , m. 99), moving to  $T_5(P)$  as "subdominant" ( $E^bM:1/5$ , m. 101), then to  $T_7(P)$  as "dominant" ( $FM:1/5$ , m. 102), and returning to "tonic" in m. 105, an octave higher than its first appearance. This structure bears a general resemblance to the tonal structure of the melody, which would center on the subdominant in m. 101 and on the dominant in m. 102. Correspondences of this type do not continue, however, after m. 105.

Other CETs appear as elaborations of the harmonic analogy. As a reflection of the usual harmonization of the melody--in the first measure, three beats of tonic and one beat of dominant--the initial statement of P is elaborated by  $C^0:1/5$  at the end of m. 99, the same embellishing pattern applied to a different quality prime collection. This category 1A pentad (SC 5-12) shares only the note F with P, as would be the case in an actual tonic-dominant relationship (although the more appropriate dominant analogue is  $T_7(P)$ , which arrives in m. 102). Due to the extensive non-intersection between the two chords, they combine to present nine different pitch-classes, arrangeable as the (non-CET) complement family surrounding a  $B^0$  triad:  $B^0:135/135 = \{B,D,F\}$  plus  $\{C,E^b,G\}$  and  $\{B^b,C^\#,E\}$ .

The second elaboration within the harmonic framework occurs in m. 102, between  $T_5(P)$  and  $T_7(P)$ . The prime collection of this elaboration is  $B^bM$ , recalling P, but the embellishing pattern (0/15) and registration are different than P. Thus this CET interrupts the move from "subdominant" (m. 101) to "dominant" (m.

102) with recollections of the prime collection of the "tonic," but not with an actual restatement that might disrupt the implied harmonic continuity. Both elaborations in the first four measures (mm. 99-102) have come from category-1 CETs-- $C^0:1/5$  from 1A,  $B^bM:0/15$  from 1B--interjecting a specificity of triadic sub-structure that is absent from the category-2 CET that is being elaborated.

The "dominant" section mm. 102-104, is elaborated first by  $F^\#M:1/5$  in m. 103. This is  $T_8(P)$ , suggesting a chromatic upper neighbor to  $T_7(P)$ , but the chord is registrally rotated so that half-step relationships are not directly apparent. Instead, the rotation brings out fifths between outer voices: in the lowest voice,  $C^\#$  (m. 103) to  $F^\#$  (m. 104), and in the highest voice,  $C$  (m. 103) to  $F$  (m. 104). An approach to the "dominant" by fifth implies a dominant preparation function, such as a "secondary dominant." The other elaboration in the "dominant" section is  $FM:3/5$  at the end of m. 104, a re-embellishment of the same prime collection as  $T_7(P)$ .

In the remainder of this excerpt, the overall growth in intensity is supported in the PDCs by dissonant enhancements of CET structures. The labels below the score in Figure 6 highlight triadic bases for many of the chords, but most of them do not qualify as CETs, and the results easily cross the boundary into "cluster" PDC types. Though legitimate CETs occur in mm. 105 ( $P, CM:0/5$ ), 106 ( $FM:1/5$ ), and 109 ( $P, B^bM:0/1$ ) more prevalent are clusters resulting from dual embellishments in mm. 106 ( $FM:1/1$ ), 107 ( $FM:1/15$ ), 110 ( $FM:1/15$ ), and 112 ( $E^bM:15/5$ ); clusters also result from the addition of notes from outside the six-tone complement family in mm. 108 and 109. M. 112 concludes with a WK cluster, and m. 113 includes dual embellishments ( $B^0:15/5$ ), one legitimate CET ( $F^\#M:5/1$ ), and apparent fist/palm chords at the end of the measure. Still, tonal references persist throughout due to the recurrence of the triadic bases from the previous passage ( $B^bM, E^bM, FM$ ), as shown in the labels. Generally, there is more emphasis at this point on the "dominant"  $FM$ , as would be

appropriate in this final preparation for the conclusion of the work (see mm. 106, 107, 110, 113).

In *Memos* (p. 104), Ives says of the *Fourth of July*, "Technically, a good deal of this movement was suggested by the old habit of piano-drum playing." The drum imitations in the excerpt above effectively illustrate the structures and roles of these chords as they had evolved from earlier practices. But Ives's comment refers to more than the imitation of a drum; he is speaking of "technique," and he cites the origins of a "good deal" of the music, certainly thinking of more than just specific "piano-drum" techniques. We can see the sense of his comment in Figure 6, in the portions where familiar CET chord-types are used for purposes other than imitating drums: the category-3 hexads within the triadic interplay (mm. 93-95), and the M:0/5 tetrads in m. 96 (left hand). Chord structures in other parts of the work are formed similarly, displaying procedures of triadic embellishment without reference to a particular "piano-drum" texture. From his piano-drum playing Ives had developed not only a new use for the piano as a source of marching rhythms and rhythmic support, but also an approach to chord construction that he found logical, coherent, and widely applicable. Much of the *Fourth of July* demonstrates the importance of the piano-drum origins in his mature style, and the effectiveness of CET structure in a broader range of musical contexts.

In view of Ives's variable working habits, musical interests, and compositional objectives, we can easily accept the diversity of his musical ideas. We do not expect a unity of musical language among separate works, or, indeed, within single works in many cases. Yet the principles of CET chord construction represent points of linkage within this diversity that can illuminate areas of broader understanding. Triadic origins are transparent, for example, in some of his ragtime imitations (e.g., *In the Inn*), and in "distortions" of tonal structures made for humorous effect or for suggesting an "echo" (e.g., *Concord Sonata*, top p. 55 of the AMP edition). And we

can also find the same set-class types used in ways that de-emphasize the triadic sub-structure. In a study of Ives's "atonal" pitch language, for example, Allen Forte cites recurrent set-classes from Ives's music that we can recognize as CETs (Fig. 5).<sup>7</sup> These include SC 4-12, used not as the CET d:0/3 or d:3/0, but as an association of a tritone and major seventh in the second movement of the *Second String Quartet* (mm. 1,3).<sup>8</sup> In *The Structure of Atonal Music*, Forte points out that the trumpet "question" in *The Unanswered Question* is SC 5-10 or 5-12 (depending on the variable last note), which we can identify as category 1A pentads on the CET list.<sup>9</sup> Both versions of this familiar five-note tune begin with the notes of a diminished triad and conclude with two embellishments (0/35 or 1/5), while avoiding pitch-class intersections with the simultaneous GM chord sustained in the strings--another application of the complementation principle.<sup>10</sup>

The opening of the *Second String Quartet* in Figure 7 will further illustrate the applicability of these ideas outside of a CET context. Portraying a metaphorical "discussion," the work opens with a motivic dialogue among the instruments, including "comments" by each participant in mm. 1-6, 7-8, and 10-12. Each section of commentary is initiated by motive "Q" in mm. 1 (viola), 7 (viola, T<sub>3</sub>), and 10-11 (2nd violin, T<sub>1</sub>I). From the opening sonority, it is apparent that the combination of a triad with its chromatic embellishments will be a central idea, perhaps the "topic" of discussion. The first chord is a Cm triad in violins and cello plus F<sup>#</sup>

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<sup>7</sup> Allen Forte, "Ives and Atonality," *An Ives Celebration: Papers and Panels of the Ives Centennial Festival-Conference*, ed. H. Wiley Hitchcock and Vivian Perlis (Urbana: University of Illinois Press, 1977, 159-186).

<sup>8</sup> *Ibid.*, 171. Forte also locates this set type in *In the Inn* (*ibid.*). Another CET cited by Forte is SC 5-17, recognizable as CM:35/0 in the song *Nov. 2, 1920*; the author notes that this is the same set-class as the opening sonority from the third of Schoenberg's *Op. 16 Five Pieces for Orchestra* (*ibid.*, 169-170).

<sup>9</sup> Forte, *The Structure of Atonal Music*, 47.

<sup>10</sup> Actually, the last note of the 5-12 version duplicates the third of the GM triad.

Figure 7. *String Quartet No. 2*, first movement, mm. 1-13.

Cm:     /5 /3 1/             /1             3/             5/

Andante moderato

Violin I

Violin II

Viola

Violoncello

*p*

7

10

*T.I(a)*

from the first note of Q in the viola, or Cm:0/5 (2A, SC 4-18). Q then moves to a D, forming 0/3 when combined with the sustained Cm chord in the other three instruments, and then to D<sup>b</sup>, or Cm:1/0. Thus Q adds, in turn, two lower (/5, /3) and one upper (1/) embellishment to the sustained triad before stopping on F and changing the complexion of the sustained sound. Then the remaining members of the Cm complement family are emphasized as the highest pitches of the subsequent comments in the other instruments: the first violin emphasizes B (/1, beat 4 m. 22), the cello emphasizes E (3/, beat 3 m. 3), and the second violin emphasizes Ab (5/, beats 1-2 m. 5). These are highlighted within and above the score in the Figure.

As the sustained notes change through the first section of discussion, the Cm context disappears--we do not perceive each emphasized embellishment in mm. 2-5 as a member of the Cm complement family. But the individual motives continually refer to similar structures, definable in relation to the CET that most closely mirrors the topic of discussion: m:0/135, or a single, same-direction embellishment of every tone of a minor triad, the category-3 hexad SC 6-19. Q, for example, presents pitch-classes {F<sup>#</sup>, D, D<sup>b</sup>, F}, a subset of 6-19 that might be explained as the thirds (D, D<sup>b</sup>) and fifths (F<sup>#</sup>, F) of two chromatically associated minor triads (roots of B and B<sup>b</sup>, respectively). The first violin motive, including all the notes from mm. 1-6, is a full statement of SC 6-19 presented as GM:15/3 (or explainable in many other ways, such as Gm:135/0). The cello (mm. 3-4) states the chromatic hexad (SC 6-1) that summarizes the previous embellishments, totalling CM:13/35 (and omitting the fifth). The brief viola motive in mm. 4-5 includes the notes (F, E, A, C), suggesting FM:0/1 or Am:5/0 (1B, SC 4-20), another subset of 6-19. Finally, the second violin summarizes many of the previous embellishments, stating Cm:135/3 through mm. 5-6.

CET principles continue to prevail in the dialogue throughout this first section. For example, T<sub>3</sub>(Q) in m. 7 begins on



Figure 8. *String Quartet No. 2*, first movement, mm. 130-132.

The musical score is for a string quartet, measures 130-132, in C minor (Cm) and 3/5 time. The score consists of four staves. The first staff (Violin I) begins with a treble clef, a key signature of one flat (B-flat), and a common time signature (C). It features a triplet of eighth notes (F4, G4, A4) and a slur over a quarter note (B4). The second staff (Violin II) also has a treble clef and a common time signature (C), with a slur over a quarter note (B4). The third staff (Viola) has a treble clef and a common time signature (C), with a slur over a quarter note (B4). The fourth staff (Cello/Double Bass) has a bass clef and a common time signature (C), with a slur over a quarter note (B4). The score includes various musical notations such as triplets, slurs, and dynamic markings. A box highlights a section of the score in measures 131 and 132, containing a triplet of eighth notes (F4, G4, A4) and a slur over a quarter note (B4). The time signature changes from 3/5 to 3/4 in measure 132. The key signature changes from C minor to C major in measure 132, indicated by the removal of the B-flat. The score ends with a double bar line and a repeat sign.

A as  $/3$  of  $F^\#M$  ( $F^\#$  cello,  $B^b$  vln 1,  $C^\#$  vln 2) and, as in m. 1, moves to other chromatic embellishments.  $Cm:0/5$  returns on beat 2 of m. 10, and  $T_1I(Q)$  (2nd vln) includes two chord tones of  $Cm$  ( $G$  and  $C$ ) and two chromatic embellishments ( $B = /1$  and  $A^b = 5/$ ). In mm. 11-12, the cello recalls SC 6-19 as  $CM:135/0$ , unfolding the notes of  $CM$  and  $C^\#M$ . Beginning an area of transition, m. 13 stabilizes briefly on  $Gm:0/1$  (beat 3), implying a "dominant" relationship with the opening sonority that had arrived from a suggestion of "subdominant" on the  $F$  in m. 12.

After m. 13, the discussion turns to other topics, including additional tonal allusions, passages of free atonality, and a quodlibet of fragments of familiar tunes. Ultimately, however, the original topics return, and the movement concludes by recalling the original chromatic embellishments. Figure 8 gives the final three measures of the movement, showing the return of  $Cm:0/1$  on the fourth beat of m. 130 and of  $Q$  in the viola in m. 131. As before,  $Q$  initiates recalls of upper and lower embellishments; now, however, they are stated concisely and without additional motivic material. After  $F^\#$  ( $/5$ ) and  $D$  ( $/3$ ) in  $Q$ , the third lower embellishment ( $B/1$ ) arrives at the end of the first violin part. The upper embellishments begin with the  $D^b$  ( $1/$ ) in  $Q$  and include the  $A^b$  ( $5/$ ) in the second violin and, finally, the  $E$  ( $3/$ ) in the viola, resembling the resolution of a 4-3 suspension to a major-quality final sonority. Apparently, the discussions have, at least for the moment, reached a point of concord.

This linkage of pitch structures in the Quartet with the dissimilar context of most music involving PDCs inspires further connections within a broader perspective of Ives's music. Embodied in CET construction is the formation of complete triads above and below a "prime collection," from which embellishments may be selected. These components are readily visible in the triadic interplay of the *Fourth of July* excerpt above (Fig. 6), and are integral to the structures in the Quartet excerpt. But rather than ascribing

greater importance to three of these nine notes, we may see the entire assemblage as a juxtaposition of equivalent structures, a "stacking" of same-quality triads that need not be limited to three stacked units. We might represent such a grouping as a cyclic repetition of the two intervals adjacent within the triad plus the interval from the fifth to the root of the triad above. To re-interpret the Cm:135/135 conceptualized in the Quartet, for example, the intervals <3,4,6> would repeat cyclically:

	Bm				Cm				C <sup>#</sup> m			
	... B	D	F <sup>#</sup>		C	E <sup>b</sup>	G		C <sup>#</sup>	E	G <sup>#</sup>	...
int. ...	3	4	6		3	4	6		3	4	6	...

Ives devoted considerable energy to experimentation with cycles of single intervals and cycles that alternate two intervals.<sup>11</sup> In the cyclic underpinning of the triad stacking we have the type of conceptual connection that could establish relationships between diverse musical creations, including "experimental" works relying on compositional calculations as well as music that is not so technically planned.

The idea of "stacking" itself has deep roots in Ives's technique and style. The concept may apply to units of stacking other than triads, and to distances other than half-steps, implying applicability of a pitch theory that describes combinations of transpositional equivalences as developed by Richard Cohn.<sup>12</sup> More broadly, we can recognize these specific aspects of pitch language reflected in general stylistic features of layering and juxtaposition, including effects of metric stratification, spatial opposition of performing forces, and tonal layering. Indeed, the *Fourth of July*

<sup>11</sup>Ives's efforts in this area are documented in J. P. Lambert, "Interval Cycles as Compositional Resources in the Music of Charles Ives," *Music Theory Spectrum*, forthcoming.

<sup>12</sup>Richard Cohn, "Inversional Symmetry and Transpositional Combination in Bartok," *Music Theory Spectrum* 10 (1988):19-42.

clearly exemplifies many of these features, so that the PDC passage, with its literal and conceptual "stackings," is itself a cross-section of a complex stratified texture. With these ingredients for a broadly based approach to diverse elements of Ives's music, we can finally return to the roots of Ives's musical ideals and philosophies, which are, by Ives's account, in his early training with his father. As examples of this influence Ives cites events involving his father that inspired programmatic musical interpretations, and philosophical positions that had been handed down from father to son. In this respect, the origins of the piano-drum chords, like other early musical experiments, stem from the same source as the technical roots of Ives's mature craft.