Structural Levels and Choice of Beat-Class Sets in Steve Reich's Phase-Shifting Music

Roberto Antonio Saltini

In his 1968 essay "Music as a Gradual Process," Steve Reich discussed the ideas underlying his compositional technique of phase-shifting,[1] which was to characterize his music from 1965 to 1971.[2] Aside from this essay by the composer himself, very few studies of Reich's phase-shifting music have appeared.[3]

From "Music as a Gradual Process," one can summarize Reich's main concerns as follows: (1) The musical structure must be clear, as in compositions where structure (in Reich's terms, "process") and musical content are identical. There can be no "hidden" structures which, according to Reich, serve only to obscure the musical process. (2) Once the musical process is set into motion, it has a life of its own and therefore needs no further interference from the composer. (3) Improvisation plays no part in the musical process, since most of the musical parameters created by improvisation cannot be easily identified. (4) No matter how objective and

---

[1]Steve Reich, *Writings about Music* (Halifax: Press of Nova Scotia College of Arts and Design, 1974), 9-11.

[2]For a comprehensive list of Reich's works which employ the phase-shifting technique, see Reich, 73-75. The date boundaries were established by Reich himself in a later essay, "Notes on Composition, 1965-1973" (Reich, 49-71).

[3]Indeed, very few analytical studies of any of Reich's compositions have appeared. For a representative sampling, though somewhat dated, of the more important European articles, see K. Robert Schwarz, "Steve Reich: Music as a Gradual Process," Part 1, *Perspectives of New Music* 19 (1980):390. See also Richard Cohn, "Transpositional Combination of Beat-Class Sets in Steve Reich's Phase-Shifting Music," *Perspectives of New Music* 30/2 (1992):146-77. Cohn not only provides an extensive bibliography of studies on Reich's music (mostly in English), but he also presents insightful analyses of *Phase Patterns* and *Violin Phase*.

controlled the process is, unexpected events will still occur in the form of "resulting patterns" (another of Reich's terms).

The summary presented above implies that Reich's phase-shifting music (in an obvious reaction against both aleatoric and serial music) enables the listener to perceive the musical process "through the sounding music," or, even better, that "the compositional process and the sounding music are the same." Therefore, in his music "there are no secrets of structure that [one] can't hear."[4] Naturally, such conditions are not appealing to the musical analyst who is driven by the challenges presented by "hidden" structures in music. Indeed, most writers point out the inadequacy of analysis for phase-shifting music, and limit themselves to a mere description of process.[5] This vein of writing depicts Reich's phase-shifting music as static, or lacking in "directionality and climax," and therefore not open to analysis.[6] On the contrary, other writers see this music as another typical product of Western musical traditions, suitable for the application of traditional analytical tools.[7]

---

[4]Reich, 10.

[5]Some writers assert that the temporal structure of such pieces is not organized by beginning, development, contrast, climax, reprise, and ending; others suggest that such pieces can only be understood from an experiential point of view. For a survey of such views, see Cohn, 176.

[6]See Schwarz, 381. See also Jonathan Kramer, *The Time of Music* (New York: Schirmer Books, 1988); Michael Nyman, *Experimental Music: Cage and Beyond* (New York: Schirmer Books, 1972); Wim Mertens, *American Minimal Music* (London: Kahn & Averill, 1983); Tim Page, "Framing the River: A Minimalist Primer," *HiFi/Musical America* 31 (November 1981):64-68.

[7]See the remarks on Reich's music by Gregory Sandow, "Steve Reich: Something New," *The Village Voice* 25/10 (10 March 1980):74. For more technical discussions on Reich's music, see Paul Epstein, "Pattern Structure and Process in Steve Reich's *Piano Phase*," *The Musical Quarterly* 72 (1986):494-502; Brian Dennis, "Repetitive and Systemic Music," *The Musical Times* 115 (1974):1036-38; Robert Morris, "Generalizing Rotational Arrays," *Journal of Music Theory* 32 (1988):91-93; and Cohn, 146-77.

In the present study, I strongly agree with the latter viewpoint, and I argue that some of Reich's own terminology may present the key to connection of his phase-shifting music to the tradition of Western musical procedures. A recent discussion of Reich's phase-shifting technique may be found in Richard Cohn's article "Transpositional Combination of Beat-Class Sets in Steve Reich's Phase-Shifting Music," cited above. Since some of Cohn's ideas form the point of departure for this paper, I will summarize them here.

Cohn's interpretation of Reich's views on traditional distinctions (such as material and process, form and content) motivates his study of Reich's music.[8] According to Cohn, it is clear that for Reich materials and processes are distinct, and that the allusion to "materials running through processes" implies the presence of an agent; of course, this agent must be the composer himself. Moreover, Cohn suggests that if materials and processes are factors in a metaphorical act of communication, then the composer must be able to receive and recognize their messages. It is not just anyone who is able to participate in this act of communication; in fact, Cohn concludes that the composer is "privy to a special craft and knowledge, bordering on secrets of structure."

Since rhythm is the most important parameter in Reich's music, the first step towards an effective analysis of his compositions is the use of a precise terminology capable of reflecting the peculiarities of his special rhythmic world. It is natural to use the theoretical apparatus developed for atonal pitch-class analysis.[9] To prepare his discussion of sets and

---

[8]The excerpt reads as follows: "Material may suggest what sort of process it should be run through (content suggests form), and processes may suggest what sort of material should be run through them (form suggests content). If the shoe fits, wear it" (Reich, 9).

[9]The resemblance of pitch and rhythmic domains in the atonal music theoretical tradition may be traced back to the seminal article by Milton Babbitt, "Twelve-Tone Rhythmic Structure and the Electronic Medium," *Perspectives of New Music* 1 (1962):49-79. Among other authors dealing with this resemblance are Benjamin Boretz, "Sketch for a Musical System (Meta-variations, Part II)," *Perspectives of New Music* 8 (1970):49-111;

their transpositional combination, Cohn defines some basic terms: (1) *beat-classes* are analogous to pitch-classes. A metric cycle consists of *n* beat-classes, arranged in a *mod-n* system and labeled from 0 to *n-1* (where 0 stands for the notated downbeat); (2) *sets* of beat-classes may have properties such as invariance under certain operations, cyclic-generability, and may enter into equivalence, similarity, and inclusion relations with each other, as do pcsets in atonal theory; (3) *basic pattern* stands for the set of beat-classes which is the source of the basic material for the whole composition.

In his analyses of *Phase Patterns* and *Violin Phase*, Cohn focuses primarily on the sets of beat-classes consisting of the union of all beat-classes attacked when the basic pattern is presented in canon with itself at different time intervals. It is Cohn's assumption that the listening experience is strongly shaped by this set. He also explores the sets consisting of beat-classes attacked simultaneously by the two canonic lines in various presentations of the basic pattern. Cohn suggests that these two sets—the Union and Intersection sets—provide two distinct experiences for the listener attuned to variations in attack-point density. The varying cardinalities of the Union and Intersection sets over the course of a composition imply two distinct formal plans, and the designs resulting from both approaches suggest that the composer was aware of the inherent properties of the beat-class sets. These observations, according to Cohn, seem to reveal a dynamic and teleological component in this music, resulting from careful choice of attack-point designs. Obviously, this has strong implications regarding the role of craft and "secrets of structure."

As Cohn points out, both *Phase Patterns* and *Violin Phase* have beat-class sets that can be generated by the smallest number greater than 1 and coprime with the cardinality of the modular universe. The presence of this

---

John Rahn, "On Pitch or Rhythm: Interpretations of Orderings of and in Pitch and Time," *Perspectives of New Music* 13 (1975):182-203; and David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987).

relation implies a further integration of the materials at a micro-structural level.[10] Another important aspect pointed out by Cohn is the presence of the *deep scale* property, as defined by Carlton Gamer.[11] This property supports maximum variety across the various transpositional levels: it implies that, for each transpositional level except the "tritone," the cardinality of the Union set of beat-classes is distinct. Finally, in light of the results obtained in his explorations, Cohn proposes a new aesthetic evaluation of Reich's phase-shifting music—an evaluation based on the fact that this music has a goal (i.e., it is teleological) and that the knowledge necessary to understand the different possibilities arising from the phase-shifting technique is very specialized.

Drawing on Cohn's definitions and results as necessary, I will explore aspects of symmetry on three different musical levels: (1) the basic pattern; (2) the various combinations of the basic pattern with its different transpositions (hereafter, *transpositional modules*);[12] and (3) the overall design. Following a different approach from Cohn's, which is based upon transpositional combination of beat-class sets, I will attempt to show that the different kinds of beat-class sets (Union, Intersection, and *Independent* sets—consisting of the union of all beat-classes exclusively attacked by each voice interacting in each transpositional module) present, locally and globally, similar palindromic designs, and that these designs reflect the *inversional symmetry* of the basic pattern. My argument proceeds from the fact that the basic pattern is a palindromic micro-structure. The process of transposing this

---

[10]This topic is treated more formally in Richard Cohn, "Properties and Generability of Transpositionally Invariant Sets," *Journal of Music Theory* 35 (1992):1-32.

[11]Carlton Gamer, "Some Combinatorial Resources of Equal-Tempered Systems," *Journal of Music Theory* 11 (1967):32-59. Gamer defines a deep scale as a set which has unique multiplicity in its interval content—that is, each entry in its interval-class vector is unique.

[12]My transpositional modules are identical to Cohn's *regions*.

pattern against itself creates another palindromic structure, this time at an intermediate structural level—the level of each transpositional module. The *sequence* of these modules implies yet another palindromic structure based on cardinalities—a structure wholly realized in the overall design of the relatively simple *Clapping Music* (1972) and partly realized in the overall design of the more elaborate *Phase Patterns* (1970).[13] A related palindromic aspect is the fact that, when the basic pattern is cycled through all transpositional levels, not only are the cardinalities of the beat-class sets repeated in reverse after the midpoint of the entire composition, but the set classes themselves are repeated in reverse.

Clapping Music occupies an interesting position in Reich's output; it is immediately subsequent to the phase-shifting period—from 1965 to 1971, according to Reich himself. Furthermore, it does not present the extra problem of the "phase-shifting transitions," where the voices shift gradually from one "locked" state to another. *Phase Patterns*, for reasons to be discussed later in the paper, does not present a complete cycle of all transpositional levels. Instead, it suggestively stops at exactly the midpoint of the cycle. However, the analytical project of this paper requires examination of a hypothetically "complete" version of the composition—a realization of one full cycle of transpositions as implied by the half-cycle which comprises the actual score. In the course of the analysis, I will discuss similarities and contrasts among sections of the two pieces and relations among the compositional materials employed in both. Finally, I will suggest yet another perspective on the two pieces based upon the aspects of symmetry which permeate them.

---

[13] *Clapping Music* (London: Universal Editions, 1980); *Phase Patterns* (London: Universal Editions, 1980).

Material and content: features of the basic patterns

Since rhythm is the only notated musical parameter in *Clapping Music*, for two hand-clappers, it is rhythm alone which determines the structure of the composition. By analogy between the pitch and rhythmic domains, the metric cycle of *Clapping Music* presents twelve beat-classes, arranged in a modulo 12 system and labeled from 0 to 11, where 0 represents the notated downbeat. The basic pattern and its axis of symmetry are shown in Figure 1. Each transpositional module presents a measure of twelve eighth-note beats, repeated twelve times with the two voices in a particular relationship. Player one maintains the basic rhythmic pattern constantly throughout the entire piece; player two moves abruptly, after twelve repetitions, from unison to one beat ahead, then, after twelve more repetitions, to two beats ahead, and so on, until the performers are back together in unison. Figure 2 shows the transpositional modules and the beat-classes attacked by each voice of *Clapping Music*, accompanied by the cardinalities of Union, Intersection, and Independent sets, respectively, given in brackets. In this figure and hereafter, the transpositional modules are identified by labeling the particular transpositions. Since the basic pattern remains untransposed in one of the voices, the label for each module consists of T_0 followed by the index number of the interacting transposition. For example, if the module involved presents the basic pattern at T_2, in addition to T_0, the label for the module is $T_{0,2}$.[14]

The main difference between the abrupt changes of *Clapping Music* and the gradual changes characteristic of Reich's earlier phase-shifting pieces resides in the fact that the gradual phasing process allows the listener to perceive a pattern "moving away" from itself continuously, with the beats themselves growing apart, then back together. On the other hand, the abrupt changes of the later pieces create a

---

[14]This labeling system was first presented by Dan Warburton, "A Working Terminology for Minimal Music," *Intégral* 2 (1988):135-36.

Figure 1. Basic pattern of *Clapping Music* {0124579A} and its axis of symmetry

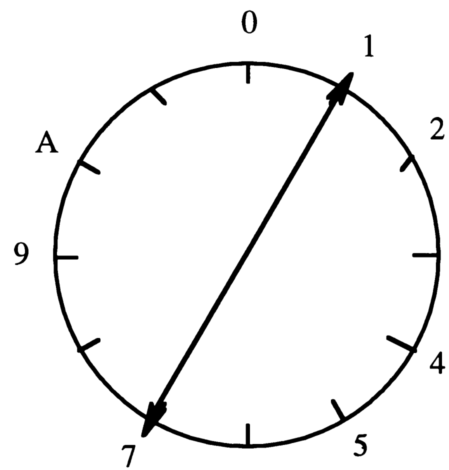
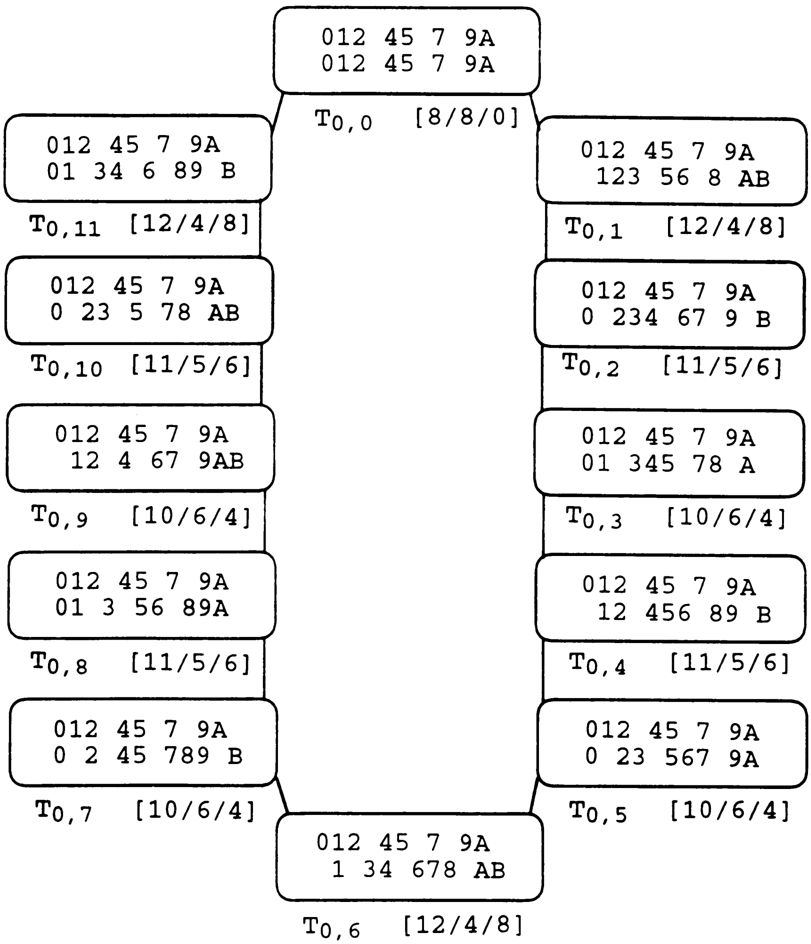


Figure 2. Transpositional modules of *Clapping Music* with the cardinalities of their Union, Intersection, and Independent sets



chain of variations based on two out-of-phase patterns with the beats always coinciding. However, this distinction between the two phasing strategies plays no part in the present study.[15]

Clapping Music uses a cycle of twelve beat-classes, eight of which are attacked in the basic pattern (Figure 1). The beat-class set (hereafter, "bc-set") formed by these attacks is {0124579A}, with interval-class vector <456562>.[16] This bc-set has several important features: (1) it is an I-symmetrical set with axis of symmetry on beats 1 and 7 in the untransposed form of the basic pattern; (2) it is a unique choice among all the possible combinations of eight attacks and four rests, given certain simple constraints to be explained below; and (3) it may be considered an "almost deep" scale.[17]

The first feature (I-symmetry) is significant because it contributes to the composition's overall unity. A corollary of John Rahn's theorem dealing with the union of I-related sets will support this point:[18]

---

[15]Paul Epstein suggests that "the phasing process is heard in several distinct stages. At first the impression is of increasing resonance, a change in acoustic quality only. At the next stage one begins to hear the voices separate: echo replaces resonance. At a certain point the irrational division of the beat caused by the echo presents a dizzying rhythmic complexity. When the voices are nearly 180°, or one half beat, out-of-phase, a doubling of the tempo is perceived; one has a momentary sense of stability, of a simplification of the irrational rhythmic relationship heard previously. This stage is very brief and is one of those events that seem to occur suddenly. The out-of-phase quality quickly returns and lasts until the new phase locks in" (Epstein, 496-97).

[16]Throughout this paper, beat-classes 10 and 11 are represented by the letters "A" and "B," respectively.

[17]An "almost deep" scale is defined as a set which results from the addition or subtraction of exactly one element from a deep scale.

[18]John Rahn, *Basic Atonal Theory* (Schirmer Books: New York, 1980), 93. The connection to Rahn's theorem, as well as the method of proof for this theorem, was suggested by John Clough.

THEOREM 1. Let S be an I-symmetrical set. Then for any n, $S \cup T_n(S)$ is also an I-symmetrical set.

Proof. The transpositions of S are not distinct from the inversions, so, for every $T_n(S)$, there exists an m such that $T_n(S) = I_m(S)$. Since, according to Rahn's theorem, $S \cup I_m(S)$ is an I-symmetrical set, it follows that $S \cup T_n(S)$ is also an I-symmetrical set.

Two other theorems, given without proof, incorporate the sets defined above as intersection and independent sets:

THEOREM 2. Let S be an I-symmetrical set. Then for any n, $S \cap T_n(S)$ is also an I-symmetrical set.

THEOREM 3. Let S be an I-symmetrical set. Then for any n, $[S \sim T_n(S)] \cup T_n(S) \sim S]$ is also an I-symmetrical set.

The theorems apply to Reich's counterpoint of two I-symmetrical sets related by transposition. Consequently, all the transpositional modules and their resulting sets present inversional symmetry. Figure 3 shows all the Union, Intersection, and Independent sets and their respective cardinalities in *Clapping Music*.

Now I consider the second feature of this particular bc-set: uniqueness. There are 495 possible arrangements of eight attacks (notes) and four rests within a twelve-beat metric cycle.[19] Of course, not all combinations are of equal musical interest; therefore, it is reasonable to propose a set of constraints under which Reich's choice is unique, assuming rotational equivalence: (1) it must begin with an attack, not a rest, to mark the beginning of the piece; (2) it must have consecutive sizes of "attack-clusters" ("note-clusters") in its

---

[19]For the mathematical formalities related to the choice of this pattern, see Joel K. Haack, "*Clapping Music*—a Combinatorial Problem," *The College Mathematics Journal* 22 (1991):224-27. My own discussion is heavily indebted to Haack's article.

Figure 3. Beat-Class Sets in *Clapping Music*

$T_{0,n}$	Union	Intersection	Independent	Cardinalities
$T_{0,0}$	{0124579A}	{0124579A}	{}	[8/8/0]
$T_{0,11}$	{0123456789AB}	{0149}	{235678AB}	[12/4/8]
$T_{0,10}$	{012345789AB}	{0257A}	{13489B}	[11/5/6]
$T_{0,9}$	{01245679AB}	{12479A}	{056B}	[10/6/4]
$T_{0,8}$	{0123456789A}	{0159A}	{234678}	[11/5/6]
$T_{0,7}$	{01245789AB}	{024579}	{18AB}	[10/6/4]
$T_{0,6}$	{0123456789AB}	{147A}	{0235689B}	[12/4/8]
$T_{0,5}$	{012345679A}	{02579A}	{1346}	[10/6/4]
$T_{0,4}$	{012456789AB}	{12459}	{0678AB}	[11/5/6]
$T_{0,3}$	{012345789A}	{01457A}	{2389}	[10/6/4]
$T_{0,2}$	{012345679AB}	{02479}	{1356AB}	[11/5/6]
$T_{0,1}$	{0123456789AB}	{125A}	{0346789B}	[12/4/8]
$T_{0,0}$	{0124579A}	{0124579A}	{}	[8/8/0]

modular system—that is, each "cluster" must be exactly one smaller or one larger than its immediate neighbors, separated by a constant number of rests. Remarkably, given eight attacks and four rests, only one bc-set (and some of its transpositions) satisfies both the above conditions, and that is Reich's choice for *Clapping Music*—{0124579A} with its progression of "attack-clusters" 3-2-1-2.

Condition (2) implies a "smoothness" characteristic—that is, a gradually increasing and decreasing number of attack-points restricted by a modular system. This bc-set, therefore, may be understood as an oscillating arithmetic progression of "attack-clusters." A step-by-step construction of Reich's set, by means of an additive combination of eighth-notes and eighth-note rests, is shown in Figure 4.[20]

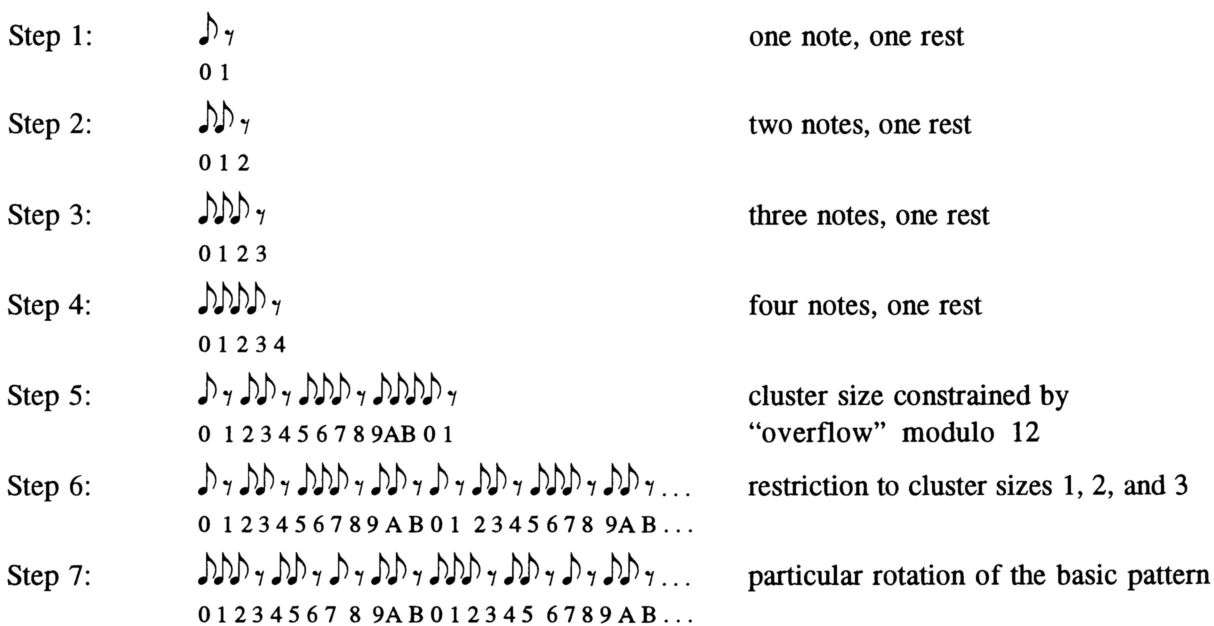
The third and last feature ("almost deepness") of this bc-set is more subtle. Jeff Pressing (1983) has shown that certain world musics present strong evidence of cognitive pitch and time isomorphisms. Following Pressing, the bc-set {0124579A} is open to a hypothetical "fusion" of its first two beat-classes, becoming bc-set {024579A}, of prime form [013568A] with interval-class vector <254361>.[21] This

---

[20]Michael Long called my attention to the fact that a similar additive process of combining rhythmic patterns is very common in the Notre Dame repertory of organa, more specifically in Perotin's music (Michael Long, "Celestial Motion and Musical Structure on the Late Middle-Ages," unpublished paper presented in February, 1993 at the University of New Mexico, Albuquerque). Curiously enough, Reich mentions his fondness of Perotin; see Donald Henahan, "Reich? Philharmonic? Paradiddling?" *New York Times*, 24 October 1971:13. It is interesting to mention that this kind of "algorithmic" generation is also present in other "minimalist" compositions, such as *1+1* by Philip Glass. This piece consists of two rhythmic cells, an eighth-note (group A) and an eighth-note plus two sixteenth-notes (group B), which must be combined in a "progressive and logical" manner, according to Glass. For instance, the sequence A, AB, ABB, ABBB, ABB, AB, A, AA, AAB, AABB, and so on, is one possible solution.

[21]Jeff Pressing, "Cognitive Isomorphisms Between Pitch and Rhythm in World Musics: West Africa, the Balkans and Western Tonality," *Studies in Music* 17 (1983):45. In this paper, Pressing equates fusion and fission with the common musical processes of "elision" and "filling-in," respectively. He uses these terms for cases where exactly one element is added or subtracted (see note 16).

Figure 4. Additive combination of “attack-clusters” and rests in basic pattern of *Clapping Music*



transformation creates the familiar diatonic set, present in many cultures including our own. It may not be a coincidence that Reich's compositions use mostly diatonic material in the pitch domain.[22] The diatonic set is a well known and thoroughly studied deep scale, and, therefore, if "fusion" is accepted as legitimate, the bc-set used by Reich in *Clapping Music* is, indeed, an "almost deep" scale. The isomorphisms between pitch and time may have even further consequences. For instance, the basic pattern of *Clapping Music*, with interval-class vector <456562>, is the only set of cardinality 8 which minimizes only interval-classes 1 and 6.[23] This observation reflects Reich's aversion to these two interval-classes, as pointed out by Cohn, and also may relate to the fact that it is virtually impossible to find more than three consecutive attack-points unseparated by a rest in any West African drum pattern.[24] Reich's interest in West African music is relevant here, and it suggests further investigation of possible connections between this music and Reich's choice of compositional materials for his phase-shifting pieces and later compositions.[25]

Drawing from the above results as necessary, I will next examine *Phase Patterns*, pointing out similarities between the basic patterns of the two pieces. In *Phase Patterns*, for four electronic organs, the basic material has interesting aspects of

---

[22]Most accounts of Reich's music relate the pitch material employed to modal collections such as dorian, mixolydian, etc. For instance, see Schwarz, "Steve Reich."

[23]Bc-set [0123578A], with interval-class vector <465472>, is also I-symmetrical; however, it minimizes interval-classes 1, 4, and 6. Bc-set [0123568A], with interval-class vector <465562>, minimizes only interval-classes 1 and 6; however, it is not I-symmetrical.

[24]Pressing, 48.

[25]For some observations about the possible influences Reich has received from his interest in West African music, see K. Robert Schwarz, "Steve Reich: Music as a Gradual Process," Part 2, *Perspectives of New Music* 20 (1981):234.

both pitch and rhythm. For comparison with *Clapping Music*, however, only the rhythmic aspects are addressed here.[26] The basic pattern of *Phase Patterns* presents a metric cycle of eight beat-classes, arranged in a modulo 8 system and labeled from 0 to 7, where 0 corresponds to the notated downbeat. The basic pattern and its axis of symmetry are shown in Figure 5. Using the basic pattern {0235} in conjunction with its transposition T_4 (i.e., {1467}), Reich presents as the basic material for the composition a particular partition known from Western rudimental drumming as a *paradiddle* pattern: LRLLRLRR, where L and R stand for left and right hands, respectively.[27]

The process of *Phase Patterns* goes as follows. The piece begins with player one presenting the paradiddle pattern about eight times. The score then shows player two beginning in unison with player one; they repeat the same paradiddle pattern many times. Next, while player one continues as before, player two gradually increases tempo and slowly moves one beat ahead of player one. This process is repeated until each player begins the pattern at exactly the halfway point of the other's pattern (i.e., player one plays LRLLRLRR against player two's RLRRLRLL). They are now 180° out of phase. During this entire process, players three and four are asked to intervene with "resulting patterns"—that is, patterns resulting from the combination of players one and two playing the same paradiddle pattern one or more beats out of phase with each other. A resulting pattern must conform to the pitch materials of players one and two; however, Reich specifies that it need not be limited to one bar in length. When players one and two reach 180° out of phase, as described above, they are doubled by players three and four, respectively. A few repetitions later, players one and two fade out, leaving players three and four alone in a kind of pedal

---

26 For an account of the pitch aspects of *Phase Patterns*, see Cohn, "Transpositional Combinations."

27 Reich, 55-56.

Figure 5. Basic pattern of *Phase Patterns* {0235} and its axis of symmetry

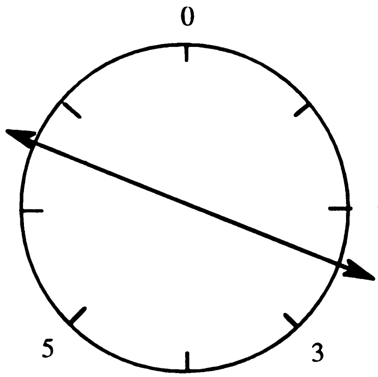
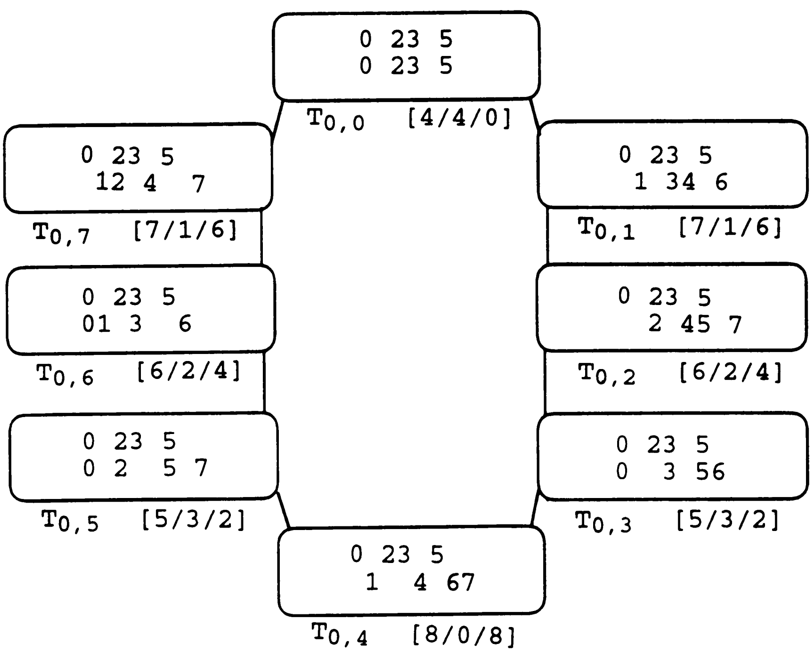


Figure 6. Composite model of *Phase Patterns* showing transpositional modules and cardinalities of their Union, Intersection, and Independent sets



point of LRLLRLRR combined with RLRRLRLL. Suddenly, without fading in, player one introduces the paradiddle pattern in rhythmic unison with player three. A few repetitions later, player two joins player one in rhythmic unison and again, while player one remains constant, player two gradually increases tempo and slowly moves one beat ahead of player one. Again this process is repeated until the players are 180° out of phase—that is, player one still playing LRLLRLRR while player two plays RLRRLRLL, in rhythmic unison with players three and four, respectively. After the required repetitions of this last bar, all four players end together.

Going back to the paradiddle pattern—that is, the combination of T_0 and T_4 of the basic pattern {0235}—it is clear that this transpositional relationship already implies a "shift" in time at a micro-structural level: the two hands of player one always play two transpositions of {0235} related by interval 4, as do the two hands of player two. In the actual music, this shift characteristic of the paradiddle pattern presents six different combinations of hands between players one and two, who, for most of the composition, realize the shifting proper. These combinations are as follows: L_1R_1, L_2R_2, L_1L_2, R_1R_2, L_1R_2, and R_1L_2.[28] The combinations L_1R_1 and L_2R_2 are considered trivial, since during the phasing process they remain the same.

Figure 6 shows a reduced version of *Phase Patterns* based on the combination of the left hands of players one and two—that is, L_1L_2. This two-hands reduction, showing the transpositional modules and the beat-classes attacked, differs from the original composition in an important detail: it continues after the $T_{0,4}$ module with $T_{0,3}$, $T_{0,2}$, $T_{0,1}$, finally regaining the unison $T_{0,0}$. This hypothetical "completion" is helpful in establishing a comparison with *Clapping Music.*

---

[28]Capital letters L an R stand for the player's left and right hands, respectively. The subscripts stand for the player's number. Therefore, L_1L_2 stands for the combination between both players one and two left hands, and L_1R_2 stands for the combination between player one left hand and player two right hand, etc.

Figure 7 shows all the Union, Intersection, and Independent sets and their respective cardinalities. This list of sets is obviously drawn from the reduced version of the composition shown in Figure 6.

Figure 8 shows, however, how this hypothetical model of a complete cycling through of the basic pattern represents the sequence of transpositional modules for the four non-trivial combinations of hands between players one and two. The reading of both schemes must be read counter-clockwise, as indicated by the arrows. The first scheme, beginning at "twelve o'clock" and "six o'clock," may represent the actual piece from the point of view of combinations L_1L_2 and L_1R_2, respectively. By the same token, the second scheme, beginning at "twelve o'clock" and "six o'clock," may represent the actual piece entirely transposed by the interval 4—that is, from the point of view of combinations R_1R_2 and R_1L_2, respectively.[29]

This sequence of transpositional modules shows, for example, that in the beginning of the composition, L_1L_2 are at $T_{0,0}$ while L_1R_2 are at $T_{0,4}$,—that is, 180° out of phase. One could say that at the beginning of the composition the phasing process is, in a certain sense, already halfway through. It follows that the end of the actual composition—which, unlike other ideal phase-shifting pieces, does not present an actual cycling through all of its transpositional levels—represents a "disguised" cycling through all of its transpositional levels. The fact that the paradiddle pattern already presents a "shift" within itself is one possible reason why Reich did not find it necessary to present a complete cycle for *Phase Patterns*.[30]

We return now to the matter of the bc-sets formed by the four attacks of each hand. These sets, {1467} and {0235}, are

---

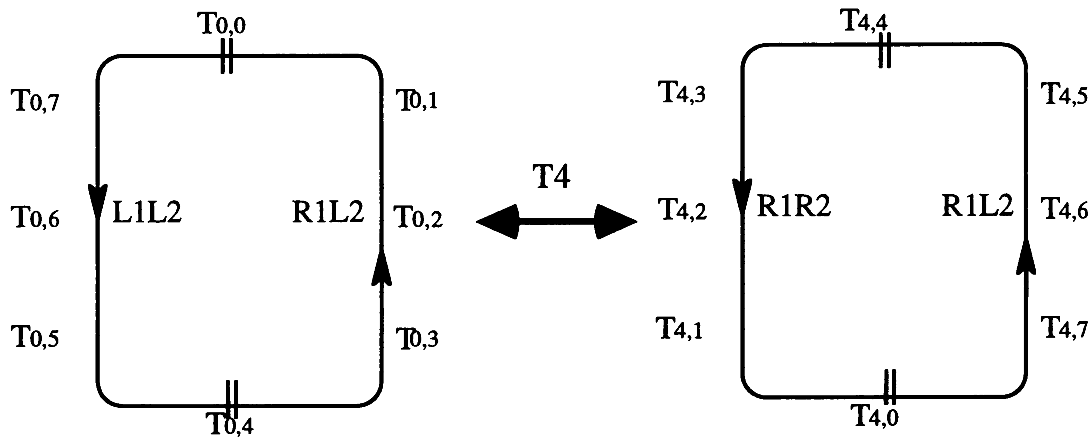
[29]John Clough called my attention to the schemes interpreting the hypothetical completion of *Phase Patterns* presented as Figure 6.

[30]This does not happen with *Clapping Music* because its basic pattern must cycle through all of its transpositional levels before it returns to its original form.

Figure 7. Beat-Class Sets in *Phase Patterns*

$T_{0,n}$	Union	Intersection	Independent	Cardinalities
$T_{0,0}$	{0235}	{0235}	{}	[4/4/0]
$T_{0,7}$	{0123457}	{2}	{013457}	[7/1/6]
$T_{0,6}$	{012356}	{03}	{1256}	[6/2/4]
$T_{0,5}$	{02357}	{025}	{37}	[5/3/2]
$T_{0,4}$	{01234567}	{}	{01234567}	[8/0/8]
$T_{0,3}$	{02356}	{035}	{26}	[5/3/2]
$T_{0,2}$	{023457}	{25}	{0347}	[6/2/4]
$T_{0,1}$	{0123456}	{3}	{012456}	[7/1/6]
$T_{0,0}$	{0235}	{0235}	{}	[4/4/0]

Figure 8. *Phase Patterns*: sequences of transpositional modules for the four non-trivial combinations of hands, players one and two



of prime form [0235] and have interval-class vector <1230>.[31] This prime form has some of the features already examined in *Clapping Music*: (1) it is an I-symmetrical set with axis of symmetry between the beat-pairs 2-3 and 6-7 in the untransposed form of the basic pattern; (2) it is a unique choice among all the possible combinations of four attacks and four rests, given certain constraints to be explained below; (3) it is a deep scale and may be considered an "almost diatonic" set. I shall discuss each of these features in detail.

Again, I-symmetry contributes to the overall unity of the composition. The theorems presented earlier apply here as well, since the two bc-sets which begin the composition already introduce the counterpoint of two I-symmetrical sets related by transposition. Moreover, the transpositional modules which follow are obviously related by theorems 1, 2, and 3—that is, they are all cases of interacting T-related I-symmetrical sets. Consequently, all the transpositional modules and their resulting sets have the same property as the basic pattern: they all present inversional symmetry.

The second feature of this bc-set, uniqueness, results from a careful choice. There are 70 possible arrangements of four attacks (notes) and four rests within an eight-beat metric cycle. As in *Clapping Music*, not all combinations are of equal musical interest; therefore, it is again reasonable to propose a set of constraints that makes Reich's choice unique, assuming rotational equivalence: (1) it must begin with an attack, not a rest, to indicate the beginning of the piece; (2) it must have consecutive sizes of "rest-clusters" in its modular system—that is, each "cluster" must be exactly one smaller or one larger than its immediate neighbors, separated by a constant number of attacks. Given four attacks and four rests, only bc-set {0235}, with "rest-cluster" sequence 0-1-2-1-0-1-2-1 . . . (one attack, no rests, one attack, one rest, one attack,

---

[31] The prime form and interval-class vector are found by adapting the conventions of atonal set theory to a modular universe of eight elements.

two rests, one attack, and so on) satisfies these conditions, and this is the set chosen by Reich in *Phase Patterns*.

Condition (2) also implies a "smoothness" characteristic, but this time it presents a gradually increasing and decreasing number of rest-points restricted by a modular system. This bc-set, too, may be understood as an oscillating arithmetic progression of "rest-clusters," following the model given for *Clapping Music*. A step-by-step construction of Reich's set, by means of an additive combination of eighth-notes and eighth-note rests, is shown in Figure 9.

The last feature of this bc-set relates again to aspects of cognitive pitch and time isomorphisms. As Cohn observes, the bc-set {0235}, with interval-class vector <1230>, is a deep scale in its modular system.[32] However, it is not a "diatonic" set in a modulo 8 system. This "problem" may be easily "corrected" by a manipulation which Pressing (1983) calls "fission." This process adds another beat-class to the original set, transforming it into the bc-set {02357}, with interval-class vector <2341>, a "diatonic" set in its modular system.[33] This suggests an affinity between pitch and rhythmic materials employed in *Phase Patterns*. The pitch-class set used throughout the entire composition is {02467B}, of prime form [013578], with interval-class vector <232341>, all modulo 12.[34] Besides being an I-symmetrical set in its modular universe, the key signature of one sharp suggests the collection {E, F♯, G, A, B, C, D} for the composition, possibly indicating the E-aeolian mode.

The previous observations regarding the bc-sets employed in both pieces should suffice to show that they

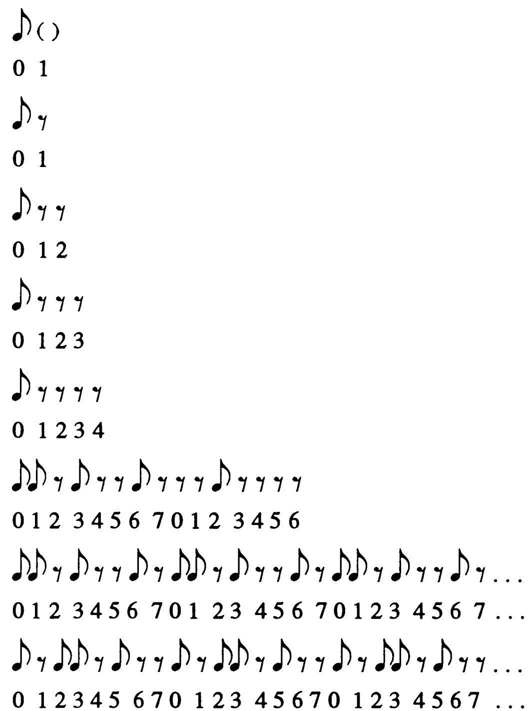
---

[32]See also Gamer, 41.

[33]The definition of "diatonic" set used here follows the one proposed by John Clough and Jack Douthett, "Maximally Even Sets," *Journal of Music Theory* 35 (1992):122-44. They characterize "diatonic" sets as those with exactly one tritone and maximal eveness in chromatic universes where the number of pcs is a multiple of 4.

[34]As a convention, I adopt pitch-class C=0.

Figure 9. Additive combination of “rest-clusters” and attacks in basic pattern of *Phase Patterns*



Step 1: one note, no rests

Step 2: one note, one rest

Step 3: one note, two rests

Step 4: one note, three rests

Step 5: one note, four rests

Step 6: cluster size constrained by “overflow” modulo 8

Step 7: restriction to cluster sizes 0, 1, and 2

Step 8: particular rotation of the basic pattern share very important features in their basic rhythmic materials and that Reich's choices may be a result of careful planning which, indeed, bares "secrets of structure." Thus far, I have discussed how the inversional symmetry present in bc-sets is reflected in a low level of structure (i.e., transpositional modules which are themselves I-symmetrical structures). Next I will attempt to show that this inversional symmetry is actually present, or at least "potentially" present, at a higher level: the level of formal design.

Form and process: Union, Intersection, and Independent attack-point sets

The main idea behind Reich's phase-shifting compositions is that they begin with the presentation of the basic pattern alone or in unison among the interacting voices. After required repetitions of the initial module, subsequent modules present the basic pattern in canon with its transpositions one beat ahead, two beats ahead, and so on. This process usually continues through a complete cycle of all transpositional modules, finally regaining the initial module to close the piece in unison. It is clear that, no matter what basic pattern is employed, the second half of such a composition is always a retrograde of the first half—that is, it presents a palindromic formal design. The midpoint of *Clapping Music* is reached at $T_{0,6}$, and the hypothetical midpoint of *Phase Patterns* is reached at $T_{0,4}$. This observation supports the claim that the low structural levels (basic pattern and transpositional modules) are closely related to the highest level of design.

As Cohn points out, there are clearly two stages involved in the phasing process: (1) the progression to the next level, which consists of a gradual or abrupt change (as in *Phase Patterns* and *Clapping Music*, respectively); and (2) a "prolongation," which consists of the sonority of each new transpositional module.[35] The prolongational sections of the two compositions produce three different bc-sets of varying attack-point density: Union sets, Intersection sets, and Independent sets, as defined previously. The three designs produced by these sets present similar results regarding the overall palindrome which constitutes the ideal form for the two compositions (see Figures 3 and 7). The exploration of Union sets, however, in most instances presents a relatively large portion of the entire modular universe. Since, throughout both compositions, each beat-class is almost always attacked by at least one player, the listening experience based on Union sets does not afford clear differentiation among the various modules. Independent sets pose a different problem. The experience of these sets closely relates to individual listening experience—that is, to the different lines one chooses to hear. It is, therefore, soft analytical ground. Moreover, Reich employs instruments of similar timbre in each of his phase-shifting compositions, thus complicating the task of grasping only those sets formed by independently attacked beat-classes.

Consequently, this section of the paper focuses on the design produced by Intersection sets. These sets are prominent throughout the compositions for a simple reason: they receive both textural and dynamic reinforcement. Intersection sets are related by way of the well-known "common-tone theorem for transposition": if one transposes a bc-set by an interval *n*, the number of common-tones between the original bc-set and its transposition equals the number of times the interval *n* occurs in the bc-set (except for the tritone, in which case the number of common tones is twice the number of tritones). Note that the cardinalities of Intersection sets, except for the first and last transpositional modules, reflect the intervallic content of the basic patterns (refer to the interval-class vectors for the basic patterns of both

[35]Following Cohn, I use the term *prolongation* to mean the sonority established by each new transpositional module.

pieces, mentioned earlier). As discussed in the first section of the paper, for complementary sections in their respective modular system, the bc-sets not only have the same cardinalities but are also equivalent under transposition. For any set S and any transpositional level n, $(S \cup T_n(S))$ and $(S \cup T_{m-n}(S))$ are equivalent under transposition, where m is the size of the universe.

Figure 2 shows the transpositional modules and the attacked beat-classes of *Clapping Music*. The cardinalities of Intersection sets are given as the middle numbers in brackets. Except for the transpositional module $T_{0,0}$, these cardinalities range from 4 to 6—4, 5, or 6 attack-points in common per bc-set. It is clear that this series progresses generally from minimum to maximum, then returns to minimum at exactly the midpoint of the piece. After that, in a mirror image of the first half, it leaps back to maximum and regresses to minimum attack-points in common.

Figure 6, the reduction of *Phase Patterns*, shows the cardinalities of the Intersection sets (the middle number in brackets, like the illustration of *Clapping Music*). Exactly as in the preceding composition, the cardinalities range from minimum to maximum, 1 to 3 attack-points in common. The only exceptions are, not surprisingly, $T_{0,0}$ and $T_{0,4}$—at the extremities and the hypothetical midpoint, respectively. As expected, the cardinalities of the entire second half are a mirror image of those in the first half.

Figures 3 and 7 support an important, though perhaps obvious, generalization about the cardinalities of composition's bc-sets: *for each prolongational module, the cardinality of the Union set equals the cardinality of the Intersection set plus the cardinality of the Independent set.*[36] This relationship effects an inverse proportion between the

---

[36]Cohn has observed an equivalent relation in *Phase Patterns*. He equates the total number of attacked beats per measure, which adds up to 8, with the series of common attacks through his regions (4, 1, 2, 3, 0) plus the series of total attacks (4, 7, 6, 5, 8).

number of common and total attack-points for every transpositional module.

In *Clapping Music*, the number of common attack-points varies from 4 to 6, and the number of total attack-points varies from 10 to 12. Transpositional modules $T_{0,11}$ and $T_{0,1}$ present twelve total attack-points and four common attack-points—maximum and minimum, respectively. The only exceptions are the first and last prolongational modules ($T_{0,0}$), which present the same number of total and common attack-points. The relationship shown above holds even for the exceptional case of $T_{0,0}$, where the cardinality of the Union set equals the sum of the cardinalities of Intersection plus Independent sets.

In *Phase Patterns*, the number of common attack-points varies from 0 to 4 and the number of total attack-points varies from 4 to 8. Transpositional modules $T_{0,7}$ and $T_{0,1}$ present seven total attack-points and one common attack-point—maximum and minimum, respectively. Again, the only exceptions are the first and last prolongational modules ($T_{0,0}$), which have the same number of common and total attack-points.

The information on cardinalities, while straightforward, reflects important references for the listener of this music. In the present study, the Intersection sets of the hypothetical completion of *Phase Patterns* (which represent at least the very obvious difference in dynamic and textural levels created by simultaneous attack-points) delineate a clear overall design: (1) no beats in common at $T_{0,4}$ as a consequence of the move from maximum ($c = 4$) to minimum ($c = 0$) variety; and (2) "potential" symmetry about $T_{0,4}$ as a consequence of the hypothetical completion of the entire cycle.[37] Cohn's study

---

[37]The symmetry here is regarded as "potential" because, as already explained, it does not occur in the actual music as a complete cycle. It is conceivable, however, that Reich realized that at the midpoint $T_{0,4} = T_{0,0}$ and decided that, in a certain sense, the cycle was completed "through the sounding music." The similarities between the bc-sets employed in *Clapping Music* and in *Phase Patterns* are, nevertheless, relevant simply because, according to Reich himself, "content suggests form"—that is,

of Union sets as the "total attack-point frequency pattern" presents a design which has similar teleological characteristics: (1) no beats in common at $T_{0,4}$ (or "aggregate completion") as a consequence of the move from minimum (c = 4) to maximum (c = 8) variety; and (2) symmetry about $T_{0,6}$ as a consequence of the adjacent interval succession presented by half of the cycle. The cardinalities of Union sets are 4, 7, 6, 5, and 8 with intervallic succession <3-1-1-3>.[38]

In summary, the findings of this portion of the paper show that both pieces present attack-point designs which may be traced back to the inner structure of their basic patterns, and that the variety obtained through the different designs (variations between maximum and minimum numbers of attack-points) indicates the possibility of the composer's desire (perhaps subconscious) to create diverse overall designs within the constraints of the phase-shifting technique.

No matter which path we choose to explore this music, it brings us to a perspective very different from the one commonly associated with Steve Reich's phase-shifting music. The composer is aware of what the process offers, and, in the traditional manner, he carefully chooses the basic pattern as a theme to be developed throughout the composition. The final product is a picture of this theme on an enlarged scale, and the path chosen to reach this picture, with its many detours, is essentially a traditional Western approach to the shaping of music.

---

palindromic bc-sets and transpositional modules may suggest an overall palindromic design. Moreover, according to Reich, "form suggests content"—that is, any bc-set would produce an overall palindromic design if cycled against all of its transpositions. However, it seems very relevant and in complete agreement with Reich's assertion that the chosen bc-sets are themselves palindromes.

[38]Cohn disregards results obtained from the "variations of common-attacks frequency" (or, simply, Intersection sets) because, according to him ". . . variations in doubling seem to be less appreciable than those in attack-point frequency" ("Transpositional Combination," 157).

References

Babbitt, Milton. 1962. "Twelve-Tone Rhythmic Structure and the Electronic Medium." *Perspectives of New Music* 1:49-79.

Boretz, Benjamin. 1970. "Sketch for a Musical System (Meta-Variations, Part II)." *Perspectives of New Music* 8:49-111.

Clough, John and Jack Douthett. 1992. "Maximally Even Sets." *Journal of Music Theory* 35:93-173.

Cohn, Richard. 1992. "Transpositional Combination of Beat-Class Sets in Steve Reich's Phase-Shifting Music." *Perspectives of New Music* 30:146-77.

________. 1992. "Properties and Generability of Transpositionally Invariant Sets." *Journal of Music Theory* 35:1-32.

Dennis, Brian. 1974. "Repetitive and Systemic Music." *The Musical Times* 115:1036-8.

Epstein, Paul. 1986. "Pattern Structure and Process in Steve Reich's *Piano Phase*." *The Musical Quarterly* 72:494-502.

Gamer, Carlton. 1967. "Some Combinatorial Resources of Equal-Tempered Systems." *Journal of Music Theory* 11:32-59.

Haack, Joel K. 1991. "Clapping Music—a Combinatorial Problem." *The College Mathematics Journal* 22:224-7.

Henahan, Donald. 1971. "Reich? Philharmonic? Paradiddling?" *New York Times* (24 October):13.

Kramer, Jonathan. 1988. *The Time of Music*. New York: Schirmer Books.

Lewin, David. 1987. *Generalized Musical Intervals and Transformations*. New Haven: Yale University Press.

Long, Michael. 1993. "Celestial Motion and Musical Structure in the Late Middle-Ages." Unpublished paper presented in February, 1993 at the University of New Mexico, Albuquerque.

Mertens, Wim. 1983. *American Minimal Music*. London: Kahn & Averill.

Morris, Robert. 1988. "Generalizing Rotational Arrays." *Journal of Music Theory* 32:75-132.

Nyman, Michael. 1972. *Experimental Music: Cage and Beyond*. New York: Schirmer Books.

Page, Tim. 1981. "Framing the River: A Minimalist Primer." *HiFi/Musical America* 31:64-8.

Pressing, Jeff. 1983. "Cognitive Isomorphisms Between Pitch and Rhythm in World Musics: West Africa, the Balkans and Western Tonality." *Studies in Music* 17:38-61.

Rahn, John. 1975. "On Pitch or Rhythm: Interpretations of Orderings of and in Pitch and Time." *Perspectives of New Music* 13:182-203.

________. 1980. *Basic Atonal Theory*. New York: Schirmer Books.

Reich, Steve. 1974. *Writings about Music*. Halifax: The Press of Nova Scotia College of Arts and Design.

Sandow, Gregory. 1980. "Steve Reich: Something New." *The Village Voice* 25/10:74.

Schwarz, K. Robert. 1980. "Steve Reich: Music as a Gradual Process," Part 1. *Perspectives of New Music* 19:373-92.

________. 1981. "Steve Reich: Music as a Gradual Process," Part 2. *Perspectives of New Music* 20:225-86.

Warburton, Dan. 1988. "A Working Terminology for Minimal Music." *Intégral* 2:135-159.